Symmetries of Symplectic Manifolds and Related Topics
Symétries des variétés symplectiques et sujets connexes
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ALEJANDRO CABRERA, Universidade Federal do Rio de Janeiro
Odd symplectic supergeometry, characteristic classes and reduction

We give an overview on the role of odd symplectic supergeometry in the description of Mathai-Quillen representatives of the Euler and Thom classes of a vector bundle. Using this lenguage, we propose natural generalizations involving (ordinary) symplectic reduction by symmetries. This is joint work with F. Bonechi.

JEFFREY CARLSON, University of Toronto
Equivariant formality beyond Hamiltonian actions

It is well known that Hamiltonian torus actions on compact symplectic manifolds are equivariantly formal; particular cases include coadjoint orbits and generalized flag manifolds G/K. Less is known in the case of the isotropy action of a Lie group K on a homogeneous space G/K when K is not of full rank in G.

In this talk I will explain the known cases and characterizations of equivariant formality of such actions in terms of ordinary cohomology, rational homotopy theory, invariant theory, and equivariant K-theory. We will also state a structure theorem for the equivariant cohomology and rationalized K-theory of such equivariantly formal actions.

Some of this work is joint with Chi-Kwong Fok.

ELISHEVA ADINA GAMSE, University of Toronto
Vanishing theorems in the cohomology ring of the moduli space of parabolic vector bundles

Let Σ be a compact connected oriented 2-manifold of genus $g \geq 2$, and let p be a point on Σ. We define a space $S_g(t)$ consisting of certain irreducible representations of the fundamental group of $\Sigma \setminus p$, modulo conjugation by $SU(n)$. This space has interpretations in algebraic geometry, gauge theory and topological quantum field theory; in particular if Σ has a Kahler structure then $S_g(t)$ is the moduli space of parabolic vector bundles of rank n over Σ.

For $n = 2$, Weitsman considered a tautological line bundle on $S_g(t)$, and proved that the $(2g)^{th}$ power of its first Chern class vanishes, as conjectured by Newstead. In this talk I will present his proof and then outline my extension of his work to $SU(n)$ and $SO(2n+1)$.

REBECCA GOLDIN, George Mason University
On equivariant structure constants for G/B

Schubert calculus concerns the product structure for rings associated with a flag manifold, G/B. For equivariant cohomology and equivariant K-theory, the coefficients are positive in an appropriate sense, reflecting underlying geometric structure. Symmetries coming from the G action lead to enumerative formulas in equivariant and ordinary cohomology and equivariant and ordinary K-theory. I will present such a formula, with a discussion of some underlying geometry. Much of this work is joint with Allen Knutson.

VICTOR GUILLEMIN, MIT
Torus actions with collinear weights

Let G be an n-torus, M a compact manifold and $G \times M \to M$ an action of G on M having the property that the fixed point sets are isolated points. For such an action the equivariant cohomology ring of M sits inside a larger ring: the "assignment ring", (a ring which describes the "orbitype stratification" of M by fixed point sets of subgroups of G), and these two rings coincide if and only if M is a GKM manifold, i.e. if and only if for every fixed point, p, the weights of the isotropy action of G on the tangent space to M at p are pairwise non-collinear. In this talk I will describe what happens when one slightly weakens this condition: i.e. requires that at most two weights be collinear.

P.S. The results I will report on are joint with Catalin Zara and Sue Tolman.

NASSER HEYDARI, Memorial University
Equivariant Perfection and Kirwan Surjectivity in Real Symplectic Geometry

Let $(M, \omega, G, \mu, \sigma, \phi)$ be a real Hamiltonian system. In this case, the real subgroup $G_{\mathbb{R}} = G^{\phi}$ acts on the real locus $Q = M^{\sigma}$. Consider an invariant inner product on the Lie algebra $\mathfrak{g}$ and define the norm squared function $f = ||\mu||^2 : M \to \mathbb{R}$. We show that under certain conditions on pairs (G, ϕ) and (M, σ), the restricted map $f_Q : Q \to \mathbb{R}$ is $G_{\mathbb{R}}$-equivariantly perfect. In particular, when the action of G on the zero level set $M_0 = f^{-1}(0)$ is free, the real Kirwan map is surjective. As an application of these results, we compute the Betti numbers of the real reduction $Q//G_{\mathbb{R}}$ of the action of the unitary group on a product of complex Grassmannian.

YAEL KARSHON, University of Toronto
Classification results in equivariant symplectic geometry

I will report on some old and new classification results in equivariant symplectic geometry, expanding on my classification, joint with Sue Tolman, of Hamiltonian torus actions with two dimensional quotients.

ALESSIA MANDINI, PUC-Rio
Symplectic embeddings and infinite staircases – Part I

McDuff and Schlenk studied an embedding capacity function, which describes when a 4-dimensional ellipsoid can symplectically embed into a 4-ball. The graph of this function includes an infinite staircase related to the odd index Fibonacci numbers. Infinite staircases have been shown to exist also in the graphs of the embedding capacity functions when the target manifold is a polydisk or the ellipsoid E(2,3).

This talk describes joint work with Cristofaro-Gardiner, Holm, and Pires, where we find new examples of symplectic toric 4-manifolds for which the graph of the embedding capacity function has an infinite staircase.

ECKHARD MEINRENKEN, University of Toronto
On the quantization of Hamiltonian loop group spaces

We will describe the construction of a spinor bundle for Hamiltonian loop group actions with proper moment maps, and various consequences. This is based on joint work with Yiannis Loizides and Yanli Song.

LEONARDO MIHALCEA, Virginia Tech
An affine quantum cohomology ring

A theorem of B. Kim identified the relations of the quantum cohomology ring of the (generalized) flag manifolds with the conserved quantities for the Toda lattice. It is expected that a similar statement exists, relating a quantum cohomology ring for the affine flag manifolds to the periodic Toda lattice. I will show how to construct a deformation of the usual quantum cohomology ring, depending on an additional affine quantum parameter. It turns out that the conserved quantities of the (dual) periodic Toda lattice give the ideal of relations in the new ring. The construction of the ring multiplication involves the "curve neighborhoods" of Schubert varieties in the affine flag manifold. For ordinary flag manifolds, these were defined and studied earlier by the speaker in several joint works with A. Buch, P.E. Chaput, and N. Perrin. This is joint with Liviu Mare.

ANA RITA PIRES, Fordham University
Symplectic embeddings and infinite staircases - Part II

This talks continues the one with the same title, on joint work with Cristofaro-Gardiner, Holm, and Mandini. I will explain the proof of the existence of infinite staircases in the graphs of the embedding capacity functions for certain symplectic toric 4-manifolds, which uses ECH capacities and Ehrhart quasipolynomials as its main tools. I will also explain why we conjecture that these are the only such manifolds for which an infinite staircase can occur.

STEVEN RAYAN, University of Saskatchewan
The quiver at the bottom of the twisted nilpotent cone on $\mathbb{CP}^1$

For the moduli space of Higgs bundles on a Riemann surface of positive genus, critical points of the natural Morse-Bott function lie along the nilpotent cone of the Hitchin fibration and are representations of A-type quivers in a twisted category of holomorphic bundles. The fixed points that globally minimize the function are representations of A_1. For twisted Higgs bundles on the projective line, the quiver describing the bottom of the cone is more complicated. We determine it and show that the moduli space is topologically connected whenever the rank and degree are coprime. This talk is based on arXiv:1609.08226.

DANIELE SEPE, Universidade Federal Fluminense
Integrable billiards and symplectic embeddings

The problem of (finding non-trivial obstructions to) embedding a symplectic manifold into another is one of the oldest in symplectic topology and started with the seminal non-squeezing theorem due to Gromov. In dimension 4, many techniques have been developed to shed light on this hard question. Recently, ECH capacities have proved effective in studying symplectic embeddings between subsets of $(\mathbb{R}^4, \omega_{\rm can})$ called toric domains, i.e. saturated with respect to the moment map of the standard Hamiltonian $\mathbb{T}^2$-action on $(\mathbb{R}^4, \omega_{\rm can})$. Motivated by work of Ramos, which uses complete integrability of the billiard on the disc to obtain some interesting embedding results for the Lagrangian bidisc by showing that the latter is symplectomorphic to a toric domain, this talk outlines how to obtain sharp obstructions to finding symplectic embeddings for some other subsets of $(\mathbb{R}^4, \omega_{\rm can})$ by relating them to suitable toric domains. These subsets are related to integrable billiards on squares and rectangles. This is ongoing joint work with Vinicius G. B. Ramos.

SHLOMO STERNBERG, Harvard University
The Stasheff associahedron

Show and tell about the Stasheff associahedron K5

JONATHAN WEITSMAN, Northeastern University
On Geometric Quantization of (some) Poisson Manifolds

Abstract: Geometric Quantization is a program of assigning to Classical mechanical systems (Symplectic manifolds and the associated Poisson algebras of C^∞ functions) their quantizations — algebras of operators on Hilbert spaces. Geometric Quantization has had many applications in Mathematics and Physics. Nevertheless the main proposition at the heart of the theory, invariance of polarization, though verified in many examples, is still not proved in any generality. This causes numerous conceptual difficulties: For example, it makes it very difficult to understand the functoriality of theory.

Nevertheless, during the past 20 years, powerful topological and geometric techniques have clarified at least some of the features of the program.

In 1995 Kontsevich showed that formal deformation quantization can be extended to Poisson manifolds. This naturally raises the question as to what one can say about Geometric Quantization in this context. In recent work with Victor Guillemin and Eva Miranda, we explored this question in the context of Poisson manifolds which are "not too far" from being symplectic—the so called b-symplectic or b-Poisson manifolds—in the presence of an Abelian symmetry group.

In this talk we review Geometric Quantization in various contexts, and discuss these developments, which end with a surprise.