Shape, Homotopy, and Attractors
Forme, homotopie et attracteurs

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SERGEY ANTONYAN, National University of Mexico (UNAM)
Characterizing G-A(N)R spaces by means of H-fixed point sets

For a Lie group G, we study equivariant extension properties of proper (in the sense of R. Palais) G-spaces. Generalizing results of J. Jaworowski and R. Lashof, we shall present in the talk characterizations of G-A(N)R spaces in terms of the H-fixed point sets, where H runs all the compact subgroups of G. Related open problems will be discussed.

ROLANDO JIMÉNEZ BENITEZ, Institute of Mathematics-UNAM-Oaxaca Branch
Free, proper and cellular actions of discrete groups on homotopy circles

Let $G \times \Sigma(1) \to \Sigma(1)$ be a free, proper and cellular action of a group G on a finite dimensional CW-complex $\Sigma(1)$ that has the homotopy type of the circle. We determine all virtually cyclic groups G that act on $\Sigma(1)$ together with the induced action $G \to \mathrm{Aut}(H^1(\Sigma(1), \mathbb{Z}))$, and we classify the orbit spaces $\Sigma(1)/G$.

This is a joint work with M. Golasinski and D. Goncalves.

FRANCISCO R. RUIZ DEL PORTAL, Universidad Complutense de Madrid
About the cohomological Conley index of isolated invariant continua

I will present the cohomological Conley index associated to an isolated invariant continuum X of a homeomorphism f and I shall discuss it in terms of the Cech cohomology groups of the unstable manifold. I'll give some consequences about the fixed point index of f at X.

(Joint work with L. Hernández-Corbato and J.J. Sánchez-Gabites.)

ALEXANDER N. DRANISHNIKOV, University of Florida
Cohomologically Strongly Infinite Dimensional Compacta

Given a coefficient ring R we define a cohomological version of strongly infinite dimensional compacta (R-SID). Compacta which are not R-SID are called cohomologically weakly infinite dimensional (R-WID). We show the R-acyclicity of the complement $Q \setminus X$ in the Hilbert cube Q of a R-WID compactum X. As a corollary we obtain the R-acyclicity of the complement results when

(a) X is weakly infinite dimensional;

(b) X has finite cohomological dimension with respect to R.

This is a joint work with A. Amarasinghe

JERZY DYDAK, University of Tennessee
Extension theorems for large scale spaces via neighborhood operators

Coarse geometry is the study of the large scale behavior of spaces. The motivation for studying such behavior comes mainly from index theory and geometric group theory. In this talk we introduce the notion of (hybrid) large scale normality for large scale spaces and prove analogues of Urysohn's Lemma and the Tietze Extension Theorem for spaces with this property, where continuous maps are replaced by (continuous and) slowly oscillating maps. To do so, we first prove a general form of each of these results in the context of a set equipped with a neighborhood operator satisfying certain axioms, from which we obtain both the classical topological results and the (hybrid) large scale results as corollaries. We prove that all metric spaces are large scale normal, and give some examples of spaces which are not hybrid large scale normal. Finally, we look at some properties of Higson coronas of a hybrid large scale normal spaces. Joint work with Thomas Weighill.

JOANNA FURNO, Indiana University-Purdue University Indianapolis
Ultrafilter constructions for group topologies

We begin by using ultrafilters to construct different topologies for profinite groups that are not strongly complete. Then we study the implications of these constructions for actions of profinite groups on Hilbert spaces. In particular, we use Hilbert space as a concrete example of a generalized cover space and find examples of groups of deck transformations with different topologies.

BORIS GOLDFARB, SUNY at Albany
Extension and non-extension theorems for coarse properties of metric spaces

There is a collection of large scale properties of metric spaces that are popular in topology related to the Novikov, Farrell-Jones, and Baum-Connes conjectures. They include finiteness of the asymptotic dimension of Gromov, finite decomposition complexity of Guentner-Tessera-Yu, asymptotic property C of Dranishnikov, property A of Yu. We generalize well-known extension theorems for these properties and prove some new results. We also show how some natural relaxation of assumptions makes the extension statements fail. We also promote the following idea: even the relaxed extension constructions allow to run variants of familiar proofs of the Novikov conjecture, in particular. From this perspective, non-extension results are very desirable because they should help to enlarge the class of groups satisfying the conjectures. This is joint work with Susan Beckhardt.

STEVE HURDER, University of Illinois at Chicago
Smooth flows with fractional entropy dimension

The fractional entropy dimension of a smooth flow, as introduced by Katok and Thouvenot, is a measure of the chaotic behavior of the flow at intermediate growth rates, between 0 and 1. A flow with positive topological entropy has entropy dimension equal to 1, while an isometric flow has entropy dimension zero. The aperiodic flows on compact 3-manifolds obtained via the celebrated construction of Krystyna Kuperberg, and called Kuperberg flows, necessarily have zero topological entropy by a theorem of Katok. In the study of the dynamics of these flows by Ana Rechtman and the presenter, it was shown that a generic Kuperberg flow has entropy dimension at least 1/2. In this work, I will show how to construct non-generic smooth Kuperberg flows which have entropy dimension 0 ¡ d ¡ 1/2, where d can be chosen arbitrarily small. We also state a conjecture relating the entropy dimension with the unstable shape properties of the unique minimal set.

DANIEL INGEBRETSON, University of Illinois at Chicago
Hausdorff dimension of Kuperberg minimal sets

The Seifert conjecture was answered negatively in 1993 by Kuperberg who constructed a smooth aperiodic flow on a three-manifold. This construction was later found to contain a minimal set with a complicated topology. This minimal set is embedded as a lamination by surfaces with a Cantor transversal of Lebesgue measure zero. In this talk we will discuss the pseudogroup dynamics on the transversal, the induced symbolic dynamics, and the Hausdorff dimension of the Cantor set.

NATALIA JONARD-PEREZ, Universidad Nacional Autónoma de México
Groups of affine transformations acting on hyperspaces of compact convex subsets of $\mathbb{R}^n$

Let $n \geq 2$. We will denote by Aff(n) the group of all affine transformations of $\mathbb{R}^n$ while $cc(\mathbb{R}^n)$ will be the hyperspace of all compact convex subsets of $\mathbb{R}^n$ equipped with the Hausdorff distance topology. In this talk we are interested in showing how the topology of certain subspaces of $cc(\mathbb{R}^n)$ is directly related to the geometry of the action of a specific subgroup of Aff(n). Understanding the dynamic of such action allows us to give a concrete description of the subspace's topology.

On the other hand, by studying the topology of the orbit spaces generated by the action of some subgroups of Aff(n) on certain subspaces of $cc(\mathbb{R}^n)$ we get some interesting results. In this line, we show that the orbit spaces $cb(\mathbb{R}^n)/$Aff(n) and $cc_1(\mathbb{R}^n)/$Sim(n) (where Sim(n) stands for the group of all similarities of $\mathbb{R}^n$) are both homeomorphic to the Banach-Mazur compactum $BM(n)$. Furthermore, if $E(n)$ denotes de Euclidean group, the orbit space $cc(\mathbb{R}^n)/E(n)$ (which corresponds with the Gromov-Hausdorff hyperspace of all compact convex subsets of $\mathbb{R}^n$) is homeomorphic to the open cone over $BM(n)$.

Many of the results presented in this talk were obtained in a joint work with Sergey Antonyan.

JAMES E. KEESLING, University of Florida

Spaces all of whose loops are small

There is a long history of trying to generalize covering spaces. A recent flurry of work on the subject has revived interest. There are spaces which are an obstacle in this study, namely, those spaces having the property that all loops are small. These spaces have no covering spaces. They deserve special consideration since any theory of generalized covering spaces must take them into account.

In this talk we show that if G is any group, then there is a pointed space (X, x_0) having the property that all loops are small and such that $\pi_1(X, x_0) \cong G$. The space X that we construct is not metrizable. In fact, it cannot be metrizable in general since there is no space (X, x_0) with all loops small such that X is first-countable at x_0 and with $\pi_1(X, x_0) \cong \mathbb{Z}$. On the other hand, we have other constructions of such spaces for certain groups G for which X is metrizable.

We will elaborate on these results and relate them to other work being done on generalized covering spaces.

DANUTA KOLODZIEJCZYK, Warsaw University of Technology

Cartesian Powers of Shapes of FANR's and Polyhedra

The Cartesian product Sh$(X) \times$ Sh(Y) of the shapes of compacta X and Y is defined as Sh$(X \times Y)$. Then Sh(X) and Sh(Y) are called factors of Sh$(X \times Y)$. Similarly one defines product and factors of pointed shapes of compacta, and factors in the homotopy category.

We prove that, if $(X, x) \in FANR$ and Sh$^n(X, x)$ =Sh(X, x), for some $2 \leq n \in N$, then Sh$(X, x) = 1$. This resolves positively a problem of J. Dydak, A. Kadlof, S. Nowak [3]. Furthermore, if $(X, x) \in FANR$, then (X, x) cannot be a proper factor of itself.

The same results we get for polyhedra in the homotopy category of CW-complexes. (In particular, on $ANR's$ shape and homotopy theory coincide.)

Thus, the answer to the following question of K. Borsuk [2] is positive: Is it true that if $X \in ANR$ and Sh$^n(X, x)$ =Sh(X, x), for some $2 \leq n \in N$, then $X \in AR$? An equivalent problem was also published in [1, Problem (7.13), p. 142].

Some related results and open problems in the homotopy category of CW-complexes, in the shape category of compacta, and on finitely presented groups, are also discussed.

REFERENCES

[1] K. Borsuk, Theory of Shape, Polish Scientific Publishers 59, Warsaw, 1975.

[2] K. Borsuk, Theory of Shape, Lecture Notes Series, 28. Matematisk Institut, Aarhus Universitet, Aarhus, 1971.

[3] J. Dydak, A. Kadlof, S. Nowak, Open Problems in Shape Theory, University of Warsaw, 1981.

ANA RECHTMAN, Universidad Nacional Autonoma de Mexico

Variations of the Kuperberg plug with positive topological entropy

A theorem of Katok implies that any aperiodic flow on a compact 3-manifold has zero topological entropy. Aperiodic flows can be constructed using the celebrated construction of Krystyna Kuperberg's plug. I will present a one-parameter family of plugs, containing the Kuperberg plug, plugs with simple dynamics (in the sense that the maximal invariant set is a cylinder) and plugs whose flow has positive topological entropy. This is joint work with Steve Hurder.

JOSE M. SANJURJO, Universidad Complutense de Madrid
Perturbation of global attractors and Shape Theory

We study continuous parametrized families of dissipative flows, which are those flows having a global attractor. The main motivation for this study comes from the observation that, in general, global attractors are not robust, in the sense that small perturbations of the flow can destroy their globality. We give a necessary and sufficient condition for a global attractor to be continued to a global attractor. We also study, using shape theoretical methods and the Conley index, the bifurcation global to non-global. We analyze, in particular, the case of coercive families, for which the bifurcation is originated by the creation of a non-saddle continuum with spherical shape. These results have been obtained in collaboration with Hector Barge.

EDWARD TYMCHATYN, University of Saskatchewan
Cell Structures

A graph is a discrete set equipped with a symmetric and reflexive relation. A cell structure is an inverse system of graphs with some mild convergence conditions. We showed recently that every topologically complete space can be obtained as the perfect image of the inverse limit of a cell structure and continuous functions between topologically complete spaces are induced by cell maps between cell structures. So topologically complete spaces and their continuous mappings can be obtained by taking inverse limits of systems of discrete approximations. This work may be thought of as improving on and extending Hausdorff's completion of a metric space and Gleason's work on absolutes of compact metric spaces. Traditionally topologically complete spaces and their mappings were obtained using inverse systems or resolutions of polyhedra or ANRs. We believe our work offers advantages over the traditional approaches because we work with 0-dimensional inverse limits so we can take all diagrams in our inverse systems to be commutative unlike in the traditional approaches.

Coauthor: Wojciech Debski

VESKO VALOV, Nipissing University
Homogeneous finite-dimensional metric compacta

Some properties of homogeneous finite-dimensional metric compacta, predominantly ANR's, will be provided. More specially, the local homological and cohomological properties of such spaces will be considered. The well-know questions about the existence of homogeneous finite-dimensional compact metric AR's, and the dimensional full-valuedness of homogeneous ANR's will be also discussed.