
Recent Trends in Algebraic Cycles, Algebraic K-Theory and Motives

Dernières tendances en cycles algébriques, K-théorie algébrique et motifs

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BEN ANTIEAU, UIC

Negative and homotopy K-theory of ring spectra and extensions of the theorem of the heart

Barwick proved that the K -theory of a stable infinity-category with a bounded t -structure agrees with the K -theory of its heart in non-negative degrees. Joint work with David Gepner and Jeremiah Heller extends this to an equivalence of nonconnective K -theory spectra when the heart satisfies certain finiteness conditions such as noetherianity. Applications to negative K -theory and homotopy K -theory of ring spectra are provided, which were the original motivation for our work.

NIKITA KARPENKO, University of Alberta

Chow ring of generic flag varieties

Let G be a split semisimple algebraic group over a field k and let X be the flag variety (i.e., the variety of Borel subgroups) of G twisted by a generic G -torsor. We study the conjecture that the canonical epimorphism of the Chow ring of X onto the associated graded ring of the topological filtration on the Grothendieck ring of X is an isomorphism. Since the topological filtration in this case is known to coincide with the computable gamma filtration, this conjecture indicates a way to compute the Chow ring.

MARC LEVINE, Universität Duisburg-Essen

Motivic Virtual Fundamental Classes

Let B be a reasonable base-scheme and Z a quasi-projective B -scheme. Relying on the Grothendieck 6-functor formalism for the motivic stable homotopy category, we define an object $C_{Z/B}^{st}$ in the motivic stable homotopy category $\mathrm{SH}(B)$, which we call the *intrinsic stable normal cone* of Z over B . For a motivic ring spectrum \mathcal{E} , we construct a fundamental class $[C_{Z/B}^{st}]_{\mathcal{E}}$ in $\mathcal{E}^{0,0}(C_{Z/B}^{st})$ and use this to construct for each perfect obstruction theory $\phi : E \rightarrow L_{Z/B}$ a virtual fundamental class $[Z, \phi]_{\mathcal{E}}^{vir} \in \mathcal{E}^{0,0}(\pi_{Z!} \Sigma^{E^\vee} 1_Z)$. Here $\pi_Z : Z \rightarrow B$ is the structure morphism and we assume that B is affine. There are also G -equivariant versions of these constructions for G a “tame” algebraic group over B .

Taking $B = \mathrm{Spec} k$ and $\mathcal{E} = H\mathbb{Z}$, the spectrum representing motivic cohomology, we recover the definition of the fundamental class $[C_{Z/B}] \in \mathrm{CH}_0(C_{Z/B})$ of the intrinsic normal cone $C_{Z/B}$ of Z and the virtual fundamental class $[Z, \phi]^{vir} \in \mathrm{CH}_r(Z)$, $r = \mathrm{rank} E$, as defined by Behrend-Fantechi. Taking $\mathcal{E} = EM(K_*^{MW})$, we get a virtual fundamental class $[Z, \phi]_{K_*^{MW}}^{vir} \in \tilde{\mathrm{CH}}_r(Z, \det^{-1} E)$, with $\tilde{\mathrm{CH}}$ the Chow-Witt theory of Barge-Morel and Fasel. In case $r = 0$, $\det E = \mathcal{O}_Z$, and Z projective over k , we can push this class forward to get a Grothendieck-Witt degree $\tilde{\mathrm{deg}}[Z, \phi]_{K_*^{MW}}^{vir} \in \mathrm{GW}(k)$.

JOSE PABLO PELAEZ MENALDO, IMATE, UNAM

A triangulated approach to the Bloch-Beilinson filtration

We will present an approach to the Bloch-Beilinson filtration in the context of Voevodsky’s triangulated category of motives.

KYLE ORMSBY, Reed College

Vanishing in motivic stable stems

Recent work of Röndigs-Spitzweck-Østvær sharpens the connection between the slice and Novikov spectral sequences. Using classical vanishing lines for the E_2 -page of the Adams-Novikov spectral sequence and the work of Andrews-Miller on the α_1 -periodic ANSS, I will deduce some new vanishing theorems in the bigraded homotopy groups of the η -complete motivic sphere spectrum. In particular, I will show that the m -th η -complete Milnor-Witt stem is bounded above (by an explicit piecewise linear function) when $m \equiv 1$ or $2 \pmod{4}$, and then lift this result to integral Milnor-Witt stems (where an additional constraint on m appears). This is joint work with Oliver Röndigs and Paul Arne Østvær.

DANIEL JUAN PINEDA, CCM-UNAM

On Nil groups of the quaternion group

We will describe the Nil groups of the ring $\mathbb{Z}Q_8$, the integral group ring of the quaternion group, we will give applications for the calculation of K theory groups of some infinite groups.

KIRSTEN WICKELGREN, Georgia Institute of Technology

Motivic Euler numbers and an arithmetic count of the lines on a cubic surface

A celebrated 19th century result of Cayley and Salmon is that a smooth cubic surface over the complex numbers contains exactly 27 lines. Over the real numbers, the number of lines depends on the surface, but work of Finashin-Kharlamov, Okonek-Teleman, and Segre shows that a certain signed count is always 3. We extend this count to an arbitrary field using A1-homotopy theory: we define an Euler number in the Grothendieck-Witt group for a relatively oriented algebraic vector bundle as a sum of local degrees, and then generalize the count of lines to a cubic surface over an arbitrary field. This is joint work with Jesse Leo Kass.

INNA ZAKHAREVICH, Cornell University

A derived zeta-function

Motivic measures can be thought of as homomorphisms out of the Grothendieck ring of varieties. Two well-known such measures are the Larsen–Lunts measure (over \mathbb{C}) and the Hasse–Weil zeta function (over a finite field). In this talk we will show how to lift the Hasse–Weil zeta function to a map of K -theory spectra which restricts to the usual zeta function on K_0 . As an application we will show that the Grothendieck spectrum contains nontrivial elements in the higher homotopy groups.