Representation Theory of Algebras
Théorie de la représentation des algèbres
(Org: **Ibrahim Assem** (Université de Sherbrooke), **Raymundo Bautista** (Universidad Nacional Autónoma de México), **Flavio Ulhoa Coelho** (Universidade de São Paulo), **Ralf Schiffler** (University of Connecticut) and/et **Sonia Trepode** (Universidad Nacional de Mar del Plata))

FRAUKE BLEHER, University of Iowa
Support varieties and holomorphic differentials

Suppose X is a smooth projective curve over an algebraically closed field k on which a finite group G acts faithfully over k. It is a classical problem to describe the kG-module structure of the holomorphic differentials $\mathrm{H}^0(X, \Omega^1_X)$. When the characteristic of k is a prime number p that divides $\#G$, this problem is much more difficult. I will discuss how one can use support varieties to study this problem. In particular, I will show that the non-maximal support variety of $\mathrm{H}^0(X, \Omega^1_X)$ is contained in a union of projective spaces associated to the inertia subgroups of the action of G on X.

THOMAS BRÜSTLE, Bishop's University and Université de Sherbrooke
Stability conditions and torsion classes

This is a report on joint work with David Smith and Hipolito Treffinger.

The notion of (semi-)stability has been introduced in representation theory of quivers by Schofield and King, and it was formalised in the context of abelian categories by Rudakov. The concept has re-appeared in mathematical physics as scattering diagrams, and the same wall-and-chamber structure is also studied in the work of Bridgeland.

It seems very natural to join two recent developments, the wall-and-chamber structure of scattering diagrams with the combinatorial structure of the fan associated with τ-tilting modules as described by Demonet, Iyama and Jasso. In fact, we learned that David Speyer and Hugh Thomas were independently following similar ideas.

We explain in the talk how the $\tau-$tiliting fan can be embedded into King's stability manifold: Each support τ-tilting pair (M, P) yields a chamber $C_{(M,P)}$, and one can give a complete description of the walls bordering this chamber $C_{(M,P)}$. Moreover, we associate to each chamber C a torsion class T_C.

We further introduce and study the notion of green paths, which can be seen as a continuous version of the maximal green sequences introduced by Keller in cluster theory in order to study Donaldson-Thomas invariants.

DIANE CASTONGUAY, Universidade Federal de Goiás
Polynomial recognition of cluster algebras of finite type

Cluster algebras form a class of commutative algebra, introduced at the beginning of the millennium by Fomin and Zelevinsky. They are defined constructively from a set of generating variables (cluster variables) grouped into overlapping subsets (clusters) of fixed cardinality. Since its inception, the theory of cluster algebras found applications in many areas of science, specially in mathematics. An important problem is to establish whether or not a given cluster algebra is of finite type. Using the standard definition, the problem is infeasible since it uses mutations that can lead to an infinite process. In 2006, Barot, Geiss and Zelevinsky presented an easier way to verify if a given algebra is of finite type, by testing if all chordless cycles of the graph related to the algebra are cyclically oriented and if there exists a positive quasi-Cartan companion of the skew-symmetrizable matrix related to the algebra. Based on this result, we develop an algorithm that verifies these conditions and decides whether or not a cluster algebra is of finite type in polynomial time. The second part of the algorithm is used to prove that the more general problem to decide if a matrix has a positive quasi-Cartan companion is in NP-class.

JOSÉ ANTONIO DE LA PEÑA, Instituto de Matemáticas, UNAM
Weakly non-negative quadratic forms (revisited)

Let q be a rational quadratic form in the variables $x_1, ..., x_n$. We say that q is (weakly) non-negative if for every vector y with (positive) non-negative coordinates, not all zero, we have $q(y) > 0$. Tits quadratic forms satisfying these properties are closely related to the representation type of algebras. We show some Jacobi-like criteria for weakly non-negativity and prove criteria for quadratic forms with rational coefficients (à la Cassel).

YADIRA VALDIVIESO DIAZ,
Computing homologies of algebras from surfaces

Given a compact, connected and closed Riemann surface S with boundary, a subset of marked points M of S and a triangulation $\mathbb{T}$, namely a maximal collection of non-crossing arcs with endpoints in M, it is possible to construct a finite dimensional path algebra $A_{\mathbb{T}}$, with properties compatible with cluster theory.

In this talk, we show how to compute four different homologies of algebras from surfaces, using the combinatoric and topological data of the triangulated surface $(S, M, \mathbb{T})$.

HERNÁN GIRALDO, Antioquia University
Shapes of Auslander-Reiten Triangles

Our main theorem classifies the Auslander-Reiten triangles according to properties of the morphisms involved. As a consequence, we are able to compute the mapping cone of an irreducible morphism. We finish by showing a technique for constructing the connecting component of the derived category of any tilted algebra. In particular we obtain a technique for constructing the derived category of any tilted algebra of finite representation type.

BIRGE HUISGEN-ZIMMERMANN, University of California at Santa Barbara
Irreducible components of varieties of representations

A longstanding problem asks for a classification of the irreducible components of the algebraic varieties parametrizing the representations with fixed dimension vector of a finite dimensional algebra. The case of truncated path algebras has been resolved in a sequence of papers, partly by the speaker, partly in collaborations of the speaker with Babson-Thomas, Bleher-Chinburg, Shipman, and Goodearl. We will present an overview.

DANIEL LABARDINI, Instituto de Matemáticas, UNAM
Species with potential arising from surfaces with orbifold points

Felikson-Shapiro-Tumarkin have associated skew-symmetrizable matrices to the triangulations of surfaces with orbifold points of order 2, and showed that triangulations related by a flip have matrices related by Fomin-Zelevinsky's matrix mutation. In this talk, based on joint work with Jan Geuenich, I will present a realization of Felikson-Shapiro-Tumarkin's matrices through species with potential, the combinatorial operation of flip of triangulations being compatible with mutations of species with potential.

SHIPING LIU, Universite de Sherbrooke
Auslander-Reiten components with bounded short cycles

Let A be an artin algebra. Cycles in the category $\operatorname{mod} A$ of finitely generated left A-modules have been extensively studied. In this talk, we shall present a joint work with Jinde Xu on AR-components Γ of $\operatorname{mod} A$ for which there exists a bound for the depths of the maps on short cycles passing through only modules in Γ. Our main result says that such an AR-component consists of a finite core containing all possible oriented cycles, finitely many left stable components which are predecessor-closed subquivers of tilted quotient algebras of A, and finitely many right stable components which are successor-closed subquivers of tilted quotient algebras of A. As a consequence, A is representation-finite if and only if there exists a bound for the depths of the maps on short cycles in $\operatorname{mod} A$. This includes a well known result of Ringel's saying that a representation-directed algebra is representation-finite, which was generalized later by Happel and Liu.

CHARLES PAQUETTE, University of Connecticut
Group actions on cluster categories and cluster algebras

We introduce the notion of admissible action of a group G on a quiver with potential (Q, W). This induces an action of G on the corresponding cluster category $\mathcal{C}(Q, W)$ and on the corresponding cluster algebra $A(Q)$. At the level of $\mathcal{C}(Q, W)$, this yields a G-precovering functor $F : \mathcal{C}(Q, W) \to \mathcal{C}(Q_G, W_G)$ where Q_G is the orbit quiver and W_G is the orbit potential. This functor is compatible with the Iyama-Yoshino mutation of a G-orbit of a summand of a cluster-tilting object of $\mathcal{C}(Q, W)$. At the level of the cluster algebra $A(Q)$, the action of G yields an algebra A_G of G-orbits of the cluster variables of the G-stable clusters. This gives generalized cluster algebras that are different from the ones introduced by Lam and Pylyavskyy. For cluster algebras arising from surfaces, those algebras are associated to some triangulated orbifolds. As in the classical case of an oriented Riemann surface with marked points, the algebra can be obtained by mutations that are specified by exchange polynomials. These algebras are different from the ones defined by Felikson-Shapiro-Tumarkin. A cluster character can also be defined to relate the category $\mathcal{C}(Q_G, W_G)$ to the algebra A_G.

MARIA JULIA REDONDO, Universidad Nacional del Sur
Cohomology of partial smash products

We define the partial group cohomology as the right derived functor of the functor of partial invariants, we relate this cohomology with partial derivations and with the partial augmentation ideal. Finally we show that there exists a Grothendieck spectral sequence relating cohomology of partial smash algebras with partial group cohomology and algebra cohomology.

Joint work with Edson Ribeiro Alvares and Marcelo Muniz Alves.

KHRYSTYNA SERHIYENKO, University of California, Berkeley
Mutation of Conway-Coxeter friezes

A frieze is a grid of positive integers that consists of a finite number of infinite rows satisfying the so-called diamond rule. Friezes were first studied by Conway and Coxeter in 1970's, but they gained fresh interest in the last decade in relation to cluster theory. In particular, there exists a bijection between friezes and cluster-tilted algebras of type A. An operation called mutation is the key notion in cluster theory, and we introduce mutations of friezes which are compatible with mutations of the associated cluster-tilted algebras.

This is joint work with K. Baur, E. Faber, S. Gratz, and G. Todorov.

PAMELA SUÁREZ, Universidad Nacional de Mar del Plata
On the global dimension of the endomorphism algebra of a τ-tilting module

We consider finite dimensional algebras over an algebraically closed field.

Recently, Adachi, Iyama and Reiten introduced a generalization of the classical tilting theory, called τ- tilting theory. It is known that mutation of tilting modules is not always possible to do. It depends on the choice of the indecomposable direct summands. Support τ- tilting modules can be regarded as a completion of the class of tilting modules from the point of view of mutation. The above mentioned authors showed that mutation of support τ- tilting modules is always possible. In addition, τ-tilting modules satisfy nice properties of tilting modules.

Given an algebra A of finite global dimension and B the endomorphism algebra of a tilting A-module, it is well-known that there exists a deep connection between the global dimension of A and the global dimension of B. Moreover, the global dimension of B is always finite.

Now, let A be an algebra of finite global dimension and B the endomorphism algebra of a τ-tilting A-module. A natural question is if there exists a relation between the global dimension of A and the global dimension of B. In order to give an answer to such a question we find some results that relate the global dimension of A with the global dimension of B. We show that the global dimension of B is not always finite. Moreover, in case we deal with a monomial algebra of global dimension 2, we prove that the global dimension of B is finite.

HUGH THOMAS, Université du Québec à Montréal
Nilpotent endomorphisms of quiver representations and reverse plane partitions

Let Q be a quiver, and let M be a representation of Q. What can we say about the generic behaviour of a nilpotent endomorphism of M? It turns out that this question is already interesting for type A_n Dynkin quivers, for which it connects up to reverse plane partitions, a classical notion in tableau combinatorics. For λ a partition, a reverse plane partition of shape λ is a filling of the boxes of λ by non-negative integers which is weakly increasing along both rows and columns. In 1971, Richard Stanley gave a beautiful formula for the generating function counting reverse plane partitions of fixed shape λ by weight (the sum of the entries). In 1976, Hillman and Grassl gave a bijective proof of Stanley's formula, introducing what is now called the Hillman-Grassl correspondence. We show that the Hillman-Grassl correspondence (and generalizations of it) can be understood in terms of the representation-theoretic question we started with. This is joint work with Al Garver and Becky Patrias.

JOSÉ A. VÉLEZ, Valdosta State University
Universal deformation rings of string modules over a class of self-injective special biserial algebras

Let k be an algebraically closed field, let Λ be a finite dimensional k-algebra and let V be a Λ-module whose stable endomorphism ring isomorphic to k. If Λ is self-injective, then V has a universal deformation ring $R(\Lambda, V)$, which is a complete local commutative Noetherian k-algebra with residue field k. Moreover, if Λ is further a Frobenius k-algebra, then $R(\Lambda, V)$ is stable under syzygies. We use these facts to determine the universal deformation rings of string Λ_N-modules with stable endomorphism ring is isomorphic to k, and which lie in a connected component of the stable Auslander-Reiten quiver of Λ_N containing a module with endomorphism ring isomorphic to k. Here $N \geq 1$ and Λ_N is a self-injective special biserial k-algebra whose Hochschild cohomology ring is a finitely generated k-algebra as proved by N. Snashall and R. Taillefer. This is a joint-work with Yohny Calderon-Henao, Hernan Giraldo and Ricardo Rueda-Robayo.

EMINE YILDIRIM, Université du Québec à Montréal
Periodic behavior of Auslander-Reiten translation

Assume R is the poset of positive roots of a finite root system Φ and $J(R)$ is the poset of order ideals of R. Let $D^b(J(R))$ be the bounded derived category of the incidence algebra of $J(R)$. Now, assume H is a hereditary algebra of type Φ. Let $Tor(H)$ be the poset of torsion classes and B_H be the incidence algebra of $Tor(H)$. Chapoton conjectures that there is a triangulated equivalence between the bounded derived categories $D^b(J(R))$ and $D^b(B_H)$. He also conjectures that $D^b(J(R))$ is fractionally Calabi-Yau, or in other words, some non-zero power of the Auslander-Reiten translation equals some power of the shift functor. Inspired by these conjectures, we investigate the action of Auslander-Reiten translation τ on the bounded derived category of the incidence algebra of some parabolic analogues of these posets. We show that the Auslander-Reiten translation τ acting on the corresponding Grothendieck groups (which is called Coxeter transformation in this context) has finite order.