Representations of Lie Algebras
Représentation des algèbres de Lie

(Org: **Vyacheslav Futorny** (University of São Paulo), **Michael Lau** (Laval University) and/et **Daniel K. Nakano** (University of Georgia))

GEORGIA BENKART, University of Wisconsin-Madison

A Tangled Approach to Cross Product Algebras, Their Invariants and Centralizers

An algebra V with a cross product has dimension 3 or 7. In this talk, we describe how 3-tangles can provide a basis for the space of homomorphisms from $\mathsf{V}^{\otimes n}$ to $\mathsf{V}^{\otimes m}$ which are invariant under the action of the automorphism group G of V. The group G is a special orthogonal group when dimV $= 3$ and a simple algebraic group of type G_2 when dimV $= 7$. When $m = n$, this gives a graphical description of the centralizer algebra $\mathsf{End}_{\mathsf{G}}(\mathsf{V}^{\otimes n})$, and hence also a graphical realization of the G-invariants in $\mathsf{V}^{\otimes 2n}$ equivalent to the First Fundamental Theorem of Invariant Theory. Our approach using certain properties of the cross product differs from that of Kuperberg, which derives quantum G_2-link invariants from the Jones polynomial starting from its simplest formulation in terms of the Kauffman bracket. The 3-dimensional simple Kaplansky Jordan superalgebra can be interpreted as a cross product (super)algebra, and 3-tangles can be used to obtain a graphical description of its invariants and centralizer algebras relative to the action of the special orthosymplectic group. This is joint work with A. Elduque.

BEN COX, College of Charleston

On the universal central extension of certain Krichever-Novikov algebras.

We will describe results on the center of the universal central extension of certain Krichever-Novikov algebras. In particular we will describe how various families of classical and non-classical orthogonal polynomials appear. We will also provide certain new identities of elliptic integrals. This material we will cover was obtained in joint work with V. Futorny, J. Tirao, M. S. Im, X. Gu, R. Luo, and K. Zhao

DIMITAR GRANTCHAROV, University of Texas at Arlington

Bounded weight modules of the Lie algebra of vector fields on the affine space

In this talk we will discuss weight modules of the Lie algebra W_n of vector fields on $\mathbb{C}^n$. A classification of all simple weight modules of W_2 with a uniformly bounded set of weight multiplicities is provided. To achieve this classification we introduce a new family of generalized tensor W_n-modules. This is a joint work with A. Cavaness.

REIMUNDO HELUANI, IMPA

Cohomology of vertex algebras

Given a vertex algebra V and its module M, I'll introduce a complex that computes the cohomology of V with coefficients in M. This cohomology theory shares some of the properties that the cohomology of Lie algebras with coefficients in modules satisfy (like computing central extensions, extensions of modules, external derivations, etc). This is joint work with B. Bakalov, A. De Sole, and V. Kac.

APOORVA KHARE, Stanford University

The Weyl-Kac weight formula

We provide the first positive formulas (without cancellations) for the weights of non-integrable simple highest weight modules over Kac-Moody algebras. For generic highest weights, we also present a formula for the weights that is similar to the Weyl-Kac character formula. For the remaining highest weights, the formula fails in a striking way, suggesting the existence of "multiplicity-free" Macdonald identities for affine root systems.

MIKHAIL KOTCHETOV, Memorial University of Newfoundland
Graded-simple modules via the loop construction

Twisted loop and multiloop algebras play an important role in the theory of infinite-dimensional Lie algebras. Given a grading by the cyclic group $\mathbb{Z}/m\mathbb{Z}$ on a semisimple Lie algebra, the loop construction produces a $\mathbb{Z}$-graded infinite-dimensional Lie algebra.

This construction was generalized by Allison, Berman, Faulkner and Pianzola to arbitrary nonassociative algebras and arbitrary quotients of abelian groups. In particular, their results, together with the recent classification of gradings by abelian groups on finite-dimensional simple Lie algebras over an algebraically closed field of characteristic zero, yield a classification of finite-dimensional graded-simple Lie algebras.

Mazorchuk and Zhao have recently applied an analogue of the loop construction to modules. In this talk, we will show how this leads to a classification of finite-dimensional graded-simple modules over semisimple Lie algebras with a grading. This is joint work with Alberto Elduque.

JONATHAN KUJAWA, University of Oklahoma
On Cyclotomic Schur Algebras

The original Schur-Weyl duality is between the general linear group and the symmetric group with the Schur algebra acting as mediator. There is an analagous story when the symmetric group is replaced with the wreath product of a cyclic group and the symmetric group and the Schur algebra is replaced with the cyclotomic Schur algebra. We will discuss a presentation of these algebras in the spirit of Doty-Giaquinto and a categorification in the spirit of Khovanov-Lauda-Rouquier. This is joint work with Jieru Zhu.

NICOLAS LIBEDINSKY, Universidad de Chile
The Anti-spherical category

I will introduce the anti-spherical category and explain how a "light leaves" theorem serves to prove that parabolic Kazhdan-Lusztig polynomials have non-negative coefficients. This is joint work with G. Williamson.

ADRIANO MOURA, University of Campinas
Tensor Products of Integrable Modules for Affine Algebras, Demazure Flags, and Partition Identities

The study of characters and related structural problems of representations of an affine Kac-Moody algebra $\widehat{\mathfrak{g}}$ often leads to proofs of interesting identities of combinatorial nature. In this talk, based on a joint work with D. Jakelic, we discuss the relation between two such structural problems: the one of computing multiplicities of irreducible modules in tensor products of two integrable irreducible modules of $\widehat{\mathfrak{g}}$ and that of computing multiplicities in Demazure flags of a given Demazure module. Our main result expresses the former in terms of the latter in the case that the underlying simple Lie algebra is simply laced. By combining our result in the case of $\widehat{\mathfrak{sl}}_2$ with the existing answers to the first problem, we obtain interesting partition identities.

ARTURO PIANZOLA,
Lie algebroids arising from infinite dimensional Lie theory

A classical construction of Atiyah assign to any (real or complex) Lie group G and manifold M a Lie algebroid over M. The spirit behind our work is to put Atiyah's construction within an algebraic context, replace M by a scheme X and G by a "simple" reductive group scheme G over X in the sense of Demazure-Grothendieck (such groups arise naturally in infinite dimensional Lie theory). Lie algebroids in an algebraic sense were also considered by Beilinson and Bernstein. We will explain how the present work relates to theirs. This is joint work with J. Kuttler and F. Quallbrunn.

CORNELIUS PILLEN, University of South Alabama
Lifting modules of a finite group of Lie type to its ambient algebraic group

Let G be a simple simply connected algebraic group over an algebraically closed field k of positive characteristic p. Inside G, the set of fixed points of the rth iterate of the Frobenius map form a subgroup, a finite of Lie type group, denoted by $G(p^r)$.

We are interested in the following question: Given a $kG(p^r)$-module M, does there always exist a G-module that is isomorphic to M as a $kG(p^r)$-module? A well-known result due to Robert Steinberg says that all the simple modules are obtained via restriction from G to $G(p^r)$. But in general the question has a negative answer.

This talk is a survey of known results together with several explicit SL_2 examples.

ALISTAIR SAVAGE, University of Ottawa
An equivalence between truncations of categorified quantum groups and Heisenberg categories

We will describe a simple diagrammatic 2-category $\mathcal{A}$ that yields a categorification of the principal realization of the basic representation of $\mathfrak{sl}_\infty$. The 2-category $\mathcal{A}$ is equivalent to a truncation of the Khovanov—Lauda categorified quantum group and also to a truncation of Khovanov's Heisenberg 2-category. After describing these results, we will discuss applications to actions of the 2-categories involved, the representation theory of the symmetric group, geometry, and W-algebras.

VERA SERGANOVA,
Representations of direct limits of classical Lie algebras and superalgebras

We define and study symmetric monoidal categories of representations of direct limits of classical Lie algebras and superalgebras, prove that these categories has enough injective objects and compute extension groups between simple objects. Then we discuss applications to construction of universal rigid tensor categories and categorification of translation and parabolic induction functors in the classical representation theory.

ANDREA SOLOTAR, Universidad de Buenos Aires
Some invariants of the super Jordan plane

Hochschild cohomology and its Gerstenhaber algebra structure are relevant Morita, tilting and derived invariants. Their computation requires a resolution of the algebra as a bimodule over itself. There is always a canonical resolution available, the bar resolution, useful from a theoretical point of view, but not in practice: its complexity rarely allows explicit calculations. Nichols algebras are generalizations of symmetric algebras in the context of braided tensor categories. They are fundamental objects for the classification of pointed Hopf algebras. Heckenberger classified finite-dimensional Nichols algebras of diagonal type up to isomorphism. The classification separates the Nichols algebras into three different classes. Later, Angiono described the defining relations of the Nichols algebras of Heckenberger's list.

In a joint work with Sebastián Reca, we computed the Hochschild (co)homology of $A = k\langle x, y\rangle/(x^2, y^2x - xy^2 - xyx)$ – the super Jordan plane–, when $char(k) = 0$. This algebra is the Nichols algebra $B(V(-1, 2))$, of Gelfand-Kirillov dimension 2.

Our main results are the following:

1) We give explicit bases for the Hochschild (co)homology spaces.

2) We describe the cup product and we thus see that the isomorphism between $H^{2p}(A, A)$ and $H^{2p+2}(A, A)$, where $p > 0$, is given by the multiplication by an element of $H^2(A, A)$, and similarly for the odd degrees.

3) We describe the Lie algebra structure of $H^1(A, A)$, which is isomorphic to a Lie subalgebra of the Virasoro algebra.

KAIMING ZHAO, Wilfrid Laurier University
Simple Witt modules that are $U(\mathfrak{h})$-free modules of finite rank

Let W_d be the Witt algebra, that is, the derivation Lie algebra of the Laurent polynomial algebra $A_d = C[x_1^{\pm 1}, x_2^{\pm 1}, ..., x_d^{\pm 1}]$. Let $\mathfrak{h}$ be the Cartan subalgeba of W_d. We will determine simple W_d-modules that are $U(\mathfrak{h})$-free modules of finite rank.