
Quantum Walks, Open Quantum Walks, Quantum Computation and Related Topics

Marches quantiques, marches quantiques ouvertes, calcul quantique et sujets connexes

(Org: **Radhakrishnan Balu** (US Army Research Lab), **Carlos F. Lardizabal** (Federal University of Rio Grande do Sul), **Chaobin Liu** (Bowie State University, USA), **Takuya Machida** (Nihon University) and/et **Salvador E. Venegas-Andraca** (Venegas-Andraca, Tecnológico de Monterrey))

RADHAKRISHNAN BALU, US Army Research Lab

Group Theoretic Treatment of Quantum Walks

We use representation theory of groups to classify coin operators of different quantum walks and provide an unified framework. We formulate quantum walks on Plancherel decompositions that are guaranteed by Peter-Weyl theorem for compact groups satisfying axiom of second countability. We build a quantum probability space and derive standard quantum walks such as Hadamard walk, Pauli operators based quantum Bernoulli processes, and walks on angular momentum space within this framework. We also outline their asymptotic behavior for different initial states using functional central limit theorems on Fock spaces, toy for time-discrete and true for time-continuous walks respectively, in terms of conjugate Brownian motions and Poisson processes. A step of the walker described in terms of annihilation and creation operators on Fock spaces provides insights into the relation between discrete and continuous time quantum walks.

DAVID FEDER, University of Calgary

Detecting topological order in two dimensions by continuous-time quantum walk

The properties of topological systems have been the subject of intense interest in recent years, both for fundamental investigations in condensed matter physics and for their potential applications to fault-tolerant quantum computation. A priority for experimentalists is verifying that a given implementation indeed supports topological phases. In this work, we show that continuous-time quantum walks of two-component particles governed by two-dimensional spin-orbit Hamiltonians can reveal the presence of topological order. The density profile in topologically non-trivial phases displays a characteristic peak in the vicinity of the origin that is absent in trivial phases. Likewise, a kink in the mean width of the particle distribution signals the presence of a quantum phase transition. The results are expected to have immediate application to systems of ultracold atoms.

ALBERTO GRUNBAUM, uc berkeley

A generalization of Schur functions, classical random walks, unitary and open quantum walks.

I will describe some of the results in a recent manuscript with Luis Velazquez, arXiv 1702.04032

The notion of Schur function originating in complex analysis has applications to unitary quantum walks as described in joint papers with J. Bourgain, L. Velazquez, A. Werner, R. Werner and J. Wilkening. In particular it gives a dynamical interpretation for the first return amplitudes for such a walk and has a role in the study of recurrence and expected return times. Furthermore certain factorization properties of these functions correspond to compositions of simpler walks.

We show here that this approach works well for classical walks as well as open quantum walks. Our results make contact with work of J. Asboth, I. Jex, T. Kiss, Z. Kurucz, C. Lardizabal, P. Sinkovicz and M. Stefanak.

CARLOS LARDIZABAL, Universidade Federal do Rio Grande do Sul

Hitting times and total variations of open quantum walks

In this work we discuss the notion of hitting times for open quantum random walks and some applications of the mean hitting time formula obtained recently in this context. Beside discussing the connection with site recurrence, a notion of total variation is presented and we study statistical aspects related to this notion.

DEBBIE LEUNG, University of Waterloo

From embezzlement (of entanglement) to breaking any (conservation) law

We start with an example, the embezzlement of entanglement, in which the conservation of entanglement under local operations is violated (to any finite accuracy) by using an appropriate initial state. We derive from this example a generic method to manipulate quantum systems coherently despite apparent violation of conservation laws. (Partly based on joint work with Ben Toner, John Watrous, and Jesse Wang.)

TAKUYA MACHIDA, Nihon University

Quantum walk on a half line

Quantum walks have been studied in mathematics, physics, and quantum information theory since around 2000. We see a quantum walk on a half line in this talk. Updated with a unitary operation, the quantum walker moves on the half line in superposition and it is observed at a position with a probability distribution. Based on the paper [1], I will show the representation for the probability distribution and its limit distribution.

[1] T. Machida : A quantum walk on the half line with a particular initial state, Quantum Information Processing, Vol.15, No.8, pp. 3101-3119 (2016).

CHANDRASHEKAR MADIAH, The Institute of Mathematical Sciences

Accelerating quantum walks

We present a scheme to describe accelerating discrete-time quantum walk. Using the scheme, we show the effect of acceleration on the entanglement between the two walkers from different frame of references. With this we will discuss the behaviour of entanglement in relativistic quantum walks and use it to simulate few interesting dynamics in relativistic quantum mechanics and quantum field theory.

VENEGAS-ANDRACA SALVADOR, Tecnologico de Monterrey

Further Advances on Quantum PageRank

Network theory plays a key role in contemporary scientific research due to the mathematical properties of networks as well as to its wide range of applications in fields like bioinformatics, food webs, metabolic networks and the structure of Internet [1]. PageRank [2], a most famous algorithm originally designed to rank nodes on the Internet, has inspired the development of algorithmic techniques for studying social networks and protein interaction networks [3] as well as the formulation of a quantum walk-based algorithm for page ranking [4].

Quantum walks were originally developed as quantum-mechanical counterparts of classical random walks. In the early days of this cross-disciplinary research field, quantum walks were used just as a mathematical tool to develop sophisticated algorithms. Later on and in stark contrast to the algorithmic properties of classical random walks, it was proved that quantum walks constitute a universal model for quantum computation. Current research efforts in the field of quantum walks include the development of quantum algorithms. In this talk we shall present a concise review of classical and quantum PageRank [4] as well as a proposal to add the feature of incoming link importance to further versions of Quantum Page Rank. *This is a work in progress.*

References

- [1] M.E.J. Newman. Networks, an introduction. Oxford University Press (2011)
- [2] L. Page et al. Stanford University Tech Report SIDL-WP-1999-0120 (1999)
- [3] M.S. Mariani et al. Scientific Reports 5, 16181 (2015)

[4] G. D. Paparo, M. A. Martin-Delgado. *Scientific Reports* 2, 444 (2012)

BARRY C SANDERS, University of Calgary
Wavelets for quantum state generation

One exciting application of a scalable universal quantum computer would be for quantum-state generation, which is a key component of quantum algorithms for quantum simulation. We are exploring the advantages of wavelets as a representation for quantum simulation of quantum fields and many-body quantum systems. Our main focus is on quantum algorithms for quantum field theories, along the lines established by Jordan, Lee and Preskill [*Science* 336: 1130-1133 (2012)], for which initial-state preparation is a dominant computational cost of the algorithm. We express the quantum field in the wavelet representation according to the method of Brennen, Rohde, Sanders and Singh [*Physical Review A* 92: 032315 (2015)], and our wavelet-enhanced algorithm for initial-state preparation as part of the quantum-field-theory quantum algorithm has a significant computational time-cost reduction over the previous lattice-based method. We are also developing a coherent-state representation for Daubechies wavelets. Progress on these wavelet representations and their use for enhancing quantum-simulation algorithms will be reported.

LUIS VELAZQUEZ, Universidad de Zaragoza
QUANTUM WALKS: RECURRENCE & TOPOLOGICAL PHASES

A monitoring approach to recurrence in quantum walks, based on checking the return after every step, has been proposed recently. A central role is played by the generating function of first returns, which turns out to be a so called Schur function, a well known mathematical object which links this quantum topic to areas like approximation theory, harmonic analysis, operator theory, orthogonal polynomials or system theory. This has been the origin of a rich interplay which allowed for a spectral characterization of quantum recurrence, but also provided new insights into unsolved mathematical problems.

A more recent discovery connects the first return generating functions to the study of symmetry protected topological phases in quantum walks. Such generating functions codify the relevant information of the evolution operator to identify the corresponding topological phases. This reduces the computation of symmetry protected topological indices to simple finite-dimensional Linear Algebra problems, allowing for a complete classification of topological phases for arbitrary non-translation invariant coined walks and generalizations.

This talk will be a review of the above results, including the geometric and topological meaning that the first return generating functions give to certain recurrence properties, or the striking behaviour of such generating functions under certain factorizations of quantum walks, which lead to recurrence splitting rules for quantum walks.

The results presented in this talk are the result of joint collaborations with R.F. Werner, C. Cedzich, C. Stahl, T. Geib (U Hannover), A.H. Werner (U Copenhagen), F.A. Grünbaum, J. Wilkening (UC Berkeley) and J. Bourgain (IAS Princeton).

MARCOS VILLAGRA, National University of Asuncion
Quantum Algorithms for Multiobjective Optimization

In this talk we will present two approaches to tackle multiobjective optimization problems using quantum algorithms. A multiobjective optimization problem is an optimization problem with two or more objective functions that need to be optimized at the same time. Usually, these objective functions present trade-offs and there is no single optimal solution, but a set of so-called Pareto-optimal solutions.

In a first approach we will show how to turn previous algorithms for single-objective optimization into multiobjective algorithms using Grover's search method. Based on experimental results, we show how a simple quantum adaptive search strategy suffices to find approximate solutions as good as the state-of-art classical multiobjective algorithm NSGAII, with evidence of a quadratic speed-up.

In our second approach, we solve a multiobjective problem using the *Quantum Adiabatic Algorithm* of Farhi et al.(arxiv:quant-ph/0001106). Here, in contrast to our quantum adaptive search method, we are able to prove finite-time convergence of the

algorithm to a single Pareto-optimal solution, provided certain natural conditions on the underlying multiobjective problem hold. This last result is completely documented in [arXiv:1605.0315](https://arxiv.org/abs/1605.0315).