Quantitative Geometry and Topology
Géométrie et topologie quantitatives
(Org: **Mikhail Belolipetsky** (IMPA), **Alexander Nabutovsky** (University of Toronto) and/et **Shmuel Weinberger** (University of Chicago))

JAE CHOON CHA, POSTECH
Bordism, chain homotopy, and Cheeger-Gromov ρ-invariants

I will talk about quantitative results for Cheeger-Gromov L^2 ρ-invariants with some applications to complexities of 3-manifolds. Also I will discuss related quantitative approaches to 4-dimensional bordism, chain homotopy, and functorial embeddings into acyclic groups.

GREG CHAMBERS, Rice University
Monotone homotopies and sweepouts

A monotone homotopy or sweepout is one which passes through each point at most once. I will describe how to turn homotopies and sweepouts into ones that are monotone, while preserving their geometric properties. These methods have a number of interesting applications, including to metric geometry and to the existence of minimal hypersurfaces. This talk involves work in collaboration with Yevgeny Liokumovich, Regina Rotman, Arnaud de Mesmay, Tim Ophelders, and Erin Chambers.

ALEXANDER DRANISHNIKOV, University of Florida
On Topological Complexity of Nonorientable Surfaces

The topological complexity $TC(X)$ of a space X was defined by M. Farber as a numerical invariant which measures the navigational complexity of X considered as a configuartion space of a mechanical system. $TC(X)$ can be defined as the minimal k such that $X \times X$ can be covered by k open set each of which deformable into the diagonal ΔX. We recall that the LS-category $catY$ of Y is the minimal number k such that Y can be covered by k open sets each of which can be deformable into a point. Thus the equality $TC(X) = cat((X \times X)/\Delta X)$ seems to be natural for reasonable X. We show that this equality does not hold true for nonorientable surfaces X of genus ¿ 1.

TULLIA DYMARZ, University of Wisconsin, Madison
BiLipschitz equivalence of coarsely dense separated nets

All coarsely dense separated nets in a metric space are quasi-isometric to each other and to the metric space in question but they are not necessarily biLipschitz equivalent. We will survey what is known about this difference and give examples of some of the techniques used to construct counter examples. Our focus will be on nets in Lie groups and in finitely generated groups.

JOEL HASS, UC Davis
Comparing Surfaces of Genus Zero

The problem of comparing the geometric shapes of a pair of surfaces with the same topology arises in facial recognition, image processing, brain cortex analysis, protein structure analysis and computer vision. We will discuss a new method to compare the shapes of two genus-zero surfaces. The method produces a new metric on the space of piecewise-smooth genus-zero Riemannian surfaces. In addition to giving a distance between a pair of surfaces, the method also produces an optimal correspondence between them. We will show some applications to neuroscience, to the study of proteins, and to geometric morphometrics (joint with Patrice Koehl).

MATTHEW KAHLE, Ohio State University
Topological solid, liquid, and gas

We study the configuration space of n disks of unit diameter in a strip of width w. We are especially interested in the asymptotic topology as $n \to \infty$, in particular the growth of the Betti numbers.

We show that there are three distinct regimes: a solid regime where homology is trivial (except in degree 0), a liquid regime where homology is unstable and grows exponentially fast, and a gas regime where homology is stable and grows polynomially fast.

This is joint work with Bob MacPherson.

SLAVA KRUSHKAL, University of Virginia
Geometric complexity of embeddings

I will discuss several notions of geometric complexity of embeddings into Euclidean spaces, and the known bounds for various dimensions. The main focus of the talk is on embeddings of simplicial 2-complexes into 4-space. (Joint work with M. Freedman)

YEVGENIY LIOKUMOVICH, MIT, USA
Quantitative aspects of Min-Max Theory

We will describe some new results in Almgren-Pitts Min-Max Theory related to volumes of minimal hypersurfaces in Riemannian manifolds. Among other results we will talk about the proof of the Weyl law for the volume spectrum (joint work with F.C. Marques and A. Neves) that was conjectured by Gromov.

BORIS LISHAK, The University of Sydney
The space of trinagulations of compact 4-manifolds

For a compact manifold M consider the space of all simplicial isomorphism classes of triangulations of M endowed with the metric defined as the minimal number of bistellar transformations required to transform one of a pair of considered triangulations into the other. Then there exist a constant $C > 1$ such that for every m and all sufficiently large N there exist more than C^N triangulations of M with at most N simplices such that pairwise distances between them are greater than $2^{2^{\cdot^{\cdot^{\cdot^{2^N}}}}}$ (m times).

This result follows from a similar result for the space of all balanced presentations of the trivial group. ("Balanced" means that the number of generators equals to the number of relations). This space is endowed with the metric defined as the minimal number of Tietze transformations between finite presentations.

I will be describing results from a joint work with Alex Nabutovsky.

FEDOR MANIN, University of Toronto
Counting thick embeddings

Given compact manifolds M and N, how can we estimate the number of isotopy classes of embeddings $M \to N$ satisfying a constraint on geometric complexity? Of course, there is a profusion of possible answers depending on the category, the dimensions of the manifolds, and the chosen measure of complexity. We show that in codimension at least 3 and for simply connected N, the number of smooth embeddings is at most polynomial with respect to a certain C^2 bound. Unlike in the case of high codimension (the so-called metastable range) the bilipschitz constant is not sufficient to obtain any finite bound; this was remarked already by Gromov in 1978. However, it remains unclear whether our measure of complexity is the "best possible"—a notion I will attempt to define.

In the case $N = \mathbb{R}^n$, we can reframe the question in terms of thick embeddings, analogous to the study of thick knots in $\mathbb{R}^3$. Several non-equivalent definitions of thick PL embeddings were given in papers of Gromov–Guth and Freedman–Krushkal; I will discuss possible definitions in the smooth category.

This is joint work with Shmuel Weinberger.

ASSAF NAOR, Princeton University
A spectral gap precludes low-dimensional embeddings

We prove that if an n-vertex $O(1)$-expander graph embeds with average distortion D into a finite dimensional normed space X, then necessarily the dimension of X is at least $n^{c/D}$ for some universal constant $c > 0$. This is sharp up to the value of the constant c, and it improves over the previously best-known estimate $\dim(X) > c(\log n)^2/D^2$.

WALTER NEUMANN, Columbia University
Some applications of coarse metrics

I will discuss examples where coarsely determined metrics have surprisingly rigid conclusions.

ALAN REID, University of Texas at Austin
Embedding arithmetic hyperbolic manifolds

In this talk we will discuss the proof that any arithmetic hyperbolic n-manifold of simplest type can either be geodesically embedded into an arithmetic hyperbolic (n+1)-manifold or its universal mod 2 abelian cover can. This leads to new results about hyperbolic n-manifolds bounding hyperbolic (n+1)-manifolds.

REGINA ROTMAN, University of Toronto
Short geodesics on closed Riemannian manifolds

We will discuss various upper bounds for the length of periodic geodesics, geodesic loops and geodesic segments on closed Riemannian manifolds. In particular, I will talk about diameter upper bounds for the length of three shortest simple periodic geodesics on a Riemannian 2-sphere (joint with Y. Lioukumovich, A. Nabutovsky), and the recent result of N. Wu and Z. Zhu estimating the length of a shortest periodic geodesic on closed Riemannian manifolds with two-sided Ricci curvature bounds.

JING TAO, University of Oklahoma
Fine geometry of the Thurston metric

In this talk, I will describe some recent results about the fine geometry of the Thurston metric on Teichmuller space. This is joint with Anna Lenzhen, David Dumas, and Kasra Rafi.