Probability Theory
Théorie des probabilités

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ALEXANDER FRIBERGH, Université de Montréal

The ant in the labyrinth

One of the most famous open problem in random walks in random environments is to understand the behavior of a simple random walk on a critical percolation cluster, a model known as the ant in the labyrinth. I will present new results on the scaling limit for the simple random walk on the critical branching random walk in high dimension. In the light of lace expansion, we believe that the limiting behavior of this model should be universal for simple random walks on critical structures in high dimensions.

MARIA GORDINA, University of Connecticut

Couplings for hypoelliptic diffusions

Coupling is a way of constructing Markov processes with prescribed laws on the same probability space. It is known that the rate of coupling (how fast you can make two processes meet) of elliptic/Riemannian diffusions is connected to the geometry of the underlying space. In this talk we consider coupling of hypoelliptic diffusions (diffusions driven by vector fields satisfying Hormander's condition). S. Banerjee and W. Kendall constructed successful Markovian couplings for a large class of hypoelliptic diffusions. We use a non-Markovian coupling of Brownian motions on the Heisenberg group, and then use this coupling to prove analytic gradient estimates for harmonic functions for the sub-Laplacian.

This talk is based on the joint work with Sayan Banerjee and Phanuel Mariano.

TOM HUTCHCROFT, University of British Columbia

The strange geometry of high-dimensional random forests

The uniform spanning forest (USF) in the lattice $\mathbb{Z}^d$, first studied by Pemantle, is defined as a limit of uniform spanning trees in growing finite boxes. Although the USF is a limit of trees, it might not be connected- Indeed, Pemantle proved that the USF in $\mathbb{Z}^d$ is connected if and only if $d < 5$. Later, Benjamini, Kesten, Peres and Schramm extended this result, and showed that the component structure of the USF undergoes a phase transition every 4 dimensions: For dimensions d between 5 and 8 there are infinitely many trees, but any two trees are adjacent; for d between 9 and 12 this fails, but for every two trees in the USF there is an intermediary tree, adjacent to each of the them. This pattern continues, with the number of intermediary trees required increasing by 1 every 4 dimensions. In this talk, I will show that this is not the whole story, and for $d > 8$ the USF geometry undergoes a qualitative change every time the dimension increases by 1.

Joint work with Yuval Peres.

LIONEL LEVINE, Cornell University

Random Harmonic Functions

Which Cayley graphs G admit a random harmonic function with translation-invariant law that is not an almost sure constant? Such a function must have infinite first moment. We show that G admits such a random harmonic function with multivariate Cauchy marginals, if and only if G admits a nonconstant deterministic bounded harmonic function. The proof is by integrating a Cauchy white noise with respect to harmonic measure on the Poisson boundary. Joint work with Alexander Holroyd and Mathav Murugan.

JESSICA LIN, McGill University
Optimal Error Estimates in the Stochastic Homogenization for Elliptic Equations in Nondivergence Form

I will present optimal quantitative error estimates in the stochastic homogenization for uniformly elliptic equations in nondivergence form. From the point of view of probability theory, stochastic homogenization is equivalent to identifying a quenched invariance principle for random walks in a balanced random environment. The main argument relies on utilizing an Efron-Stein type concentration inequality combined with regularity estimates on the random solutions. This talk is based on joint work with Scott Armstrong.

EYAL LUBETZKY, Courant Institute, NYU
Mixing times of critical Potts models

We will discuss recent progress, jointly with R. Gheissari, on the dynamical phase transition for the critical q-state Potts model on the 2D torus (both single-site dynamics such as Glauber/Metropolis and cluster dynamics such as Swendsen-Wang), where the conjectured behavior was a mixing time that is polynomial in the side-length for $q = 2, 3, 4$ colors but exponential in it for $q > 4$. We will compare these results with the behavior on the complete graph on n vertices at critcality, sketching a proof from recent work with R. Gheissari and Y. Peres that, in that setting, Swendsen-Wang dynamics is exponentially slow in n.

SERGUEI POPOV, UNICAMP
Two-dimensional random interlacements

We define the model of two-dimensional random interlacements using simple random walk trajectories conditioned on never hitting the origin, and then obtain some its properties. Also, for random walk on a large torus conditioned on not hitting the origin up to some time proportional to the mean cover time, we show that the law of the vacant set around the origin is close to that of random interlacements at the corresponding level. Thus, this new model provides a way to understand the structure of the set of late points of the covering process from a microscopic point of view. Also, we discuss a continuous version of the model, build using the conditioned (on not hitting the unit disk) Brownian motion trajectories. This is a joint work with Francis Comets and Marina Vachkovskaia.

LEONARDO ROLLA, University of Buenos Aires
Absorbing-State Phase Transitions 3.0

Modern statistical mechanics offers a large class of driven-dissipative stochastic systems that naturally evolve to a critical state, of which Activated Random Walks are perhaps the best example. The main pursuit in this field is to show universality of critical parameters, describe the critical behavior, the scaling relations and critical exponents of such systems, and the connection between driven-dissipative dynamics and conservative dynamics in infinite space.

This problem was stuck for more than a decade, then it saw significant partial progress about 9 years ago, and got stuck again. In this talk we will report on exciting progress made in the last 4 years, thanks to the contributions of Basu, Cabezas, Ganguly, Hoffman, Sidoravicius, Stauffer, Taggi, Teixeira, Tournier, Zindy, and myself. We will also discuss some of the several open problems.

SANCHAYAN SEN, McGill University
Novel scaling limits for random discrete structures

Based on empirical arguments, statistical physicists conjectured in the early 2000s that for random graph models on n vertices with degree exponent $\tau \in (3, 4)$, typical distance both at criticality and in the strong disorder regime scales like $n^{\frac{\tau-3}{\tau-1}}$. This stands in stark contrast to the behavior of the classical Erdos-Renyi random graph model where the scaling is known to be $n^{1/3}$. Only very recently, this conjecture has been verified mathematically for a number of models. We discuss a general approach to this problem that relies on coupling random graph processes with the multiplicative coalescent, and how this method can be applied to prove the conjecture for the inhomogeneous random graph model, graphs with given degree sequence, and the configuration model.

MYKHAYLO SHKOLNIKOV, Princeton University
Edge of spiked beta ensembles

We will discuss a new approach to the study of the edge of spiked beta ensembles. The latter arise e.g. when a GOE/GUE/GSE matrix is perturbed by a low rank deterministic matrix. The limit is described by a Feynman-Kac type semigroup of random operators. Various quantities for the latter can be then computed using tools of stochastic calculus. This is joint work with Pierre Yves Gaudreau Lamarre and is based on some ideas from previous work with Vadim Gorin.

AUGUSTO TEIXEIRA, IMPA - Rio de Janeiro
Sharpness of the phase transition for continuum percolation in R^2

We study the phase transition of random radii Poisson Boolean percolation: Around each point of a planar Poisson point process, we draw a disc of random radius, independently for each point. The behavior of this process is well understood when the radii are uniformly bounded from above. In this article, we investigate this process for unbounded (and possibly heavy tailed) radii distributions. Under mild assumptions on the radius distribution, we show that both the vacant and occupied sets undergo a phase transition at the same critical parameter λ_c. Moreover,

• For $\lambda < \lambda_c$, the vacant set has a unique unbounded connected component and we give precise bounds on the one-arm probability for the occupied set, depending on the radius distribution.

• At criticality, we establish the box-crossing property, implying that no unbounded component can be found, neither in the occupied nor the vacant sets. We provide a polynomial decay for the probability of the one-arm events, under sharp conditions on the distribution of the radius.

• For $\lambda > \lambda_c$, the occupied set has a unique unbounded component and we prove that the one-arm probability for the vacant decays exponentially fast.

The techniques we develop in this article can be applied to other models such as the Poisson Voronoi and confetti percolation.

LI-CHENG TSAI, Columbia University
The Speed-N^2 Large Deviations of the TASEP

Consider the Totally Asymmetric Simple Exclusion Process (TASEP) on the integer lattice $\mathbb{Z}$. Under the scaling N^{-1} of space and time, we study the functional Large Deviations (LDs) of the integrated current. As hinted by the asymmetric of the LD of the exponential Last Passage Percolation, the TASEP exhibits two types of LDs: one corresponds to events of probability $\exp(-O(N))$, and the other corresponds to events of probability $\exp(-O(N^2))$. In this talk I will report a result on the speed-N^2 LD Principle of the TASEP, with an explicit rate function. This result complements the speed-N LD Principle of Jensen (2000) and of Varadhan (2004). Also, viewing the TASEP as a special degeneration of the stochastic Six Vertex Model, we interpret our result as giving an explicit formula of the surface tension function of a titling model.

This is joint work with Stefano Olla

FREDERI VIENS, Michigan State University
Applications of the analysis on Wiener space

Tools from the analysis on Wiener space, including Wiener chaos calculus and the Malliavin calculus, were promoted historically to help develop the theory of stochastic analysis and its applications to other parts of probability and analysis. They are becoming increasingly helpful as sharp tools for the quantitative analysis of asymptotic questions. We will present a brief introduction to these tools, how they are helping redefine and expand Stein's method for normal approximations, and how they can have important practical implications in mathematical statistics. Parts of this talk are based on joint works with Luis A. Barboza, Khalifa es-Sebaiy, and Léo Neufcourt.

TIANYI ZHENG, UC San Diego
Joint behavior of volume growth and entropy of random walks on groups

In the last few years there has been significant advancement in understanding the possible range of behaviors of the volume growth and of the entropy and rate of escape of random walks on groups. Bartholdi and Erschler constructed the first family of intermediate growth groups whose volume growth function follows any prescribed nice enough function in the exponent range $[\alpha_0, 1]$ for some explicit $\alpha_0 \approx 0.7674$. We discuss a variant of a construction of Kassabov and Pak which provides an alternative proof of the Bartholdi-Erschler result. Different behaviors of entropy of random walks on these two families of groups allow us to deduce a result concerning possible joint behavior of intermediate volume growth and entropy of random walks within a certain range of parameters. Joint with Gidi Amir.