Operator Theory on Function Spaces
Théorie des opérateurs sur des espaces de fonctions

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CATHERINE BENETEAU, University of South Florida

Zeros of optimal polynomial approximants in Dirichlet-type spaces

In this talk, I will discuss certain polynomials that are optimal approximants of inverses of functions in growth restricted analytic function spaces of the unit disk. I will examine the structure of the zeros of these optimal approximants and in particular study an extremal problem whose solution is related to Jacobi matrices and real orthogonal polynomials on the real line.

RAPHAEL CLOUATRE, University of Manitoba

Annihilating ideals and spectra for commuting row contractions

We investigate the relationship between an ideal of multipliers and the spectra of operators on Hilbert space annihilated by the ideal. This relationship is well-known and especially transparent in the single variable case, but we focus on the multivariate situation and the associated function theoretic framework of the Drury-Arveson space. Recent advances in the structure of multipliers are leveraged to obtain a description of the spectrum of a commuting row contraction annihilated by a given ideal.

LEWIS COBURN, State University of New York at Buffalo

Toeplitz Quantization

I discuss some recent work with Wolfram Bauer and Raffael Hagger. Here, C^n is complex n-space and, for z in C^n, we consider the standard family of Gaussian measures $d\mu_t(z) = (4\pi t)^{-n} exp(-|z|^2/4t)dv(z), t > 0$ where dv is Lebesgue measure. We consider the Hilbert space L^2_t of all μ_t-square integrable complex-valued measurable functions on C^n and the closed subspace of all square-integrable entire functions, H^2_t. For f measurable and h in H^2_t with fh in L^2_t, we consider the Toeplitz operators $T^{(t)}_f h = P^{(t)}(fh)$ where $P^{(t)}$ is the orthogonal projection from L^2_t onto H^2_t. For bounded f (f in L^∞) and some unbounded f, these are bounded operators with norm $|| \cdot ||_t$. For f, g bounded, with "sufficiently many" bounded derivatives, there are known deformation quantization conditions, including $(0) lim_{t\to 0} ||T^{(t)}_f||_t = ||f||_\infty$ and $(1) lim_{t\to 0} ||T^{(t)}_f T^{(t)}_g - T^{(t)}_{fg}||_t = 0$. We exhibit a pair of bounded real-analytic functions F, G so that (1) fails. On the positive side, for the space VMO of functions with vanishing mean oscillation, we show that (1) holds for all f in (the sup-norm closed algebra) $VMO \cap L^\infty$ and g in L^∞. (1) also holds for all f in UC (uniformly continuous functions, bounded or not) while (0) holds for all bounded continuous f.

RAUL CURTO, University of Iowa

A New Necessary Condition for the Hyponormality of Toeplitz Operators on the Bergman Space

A well known result of C. Cowen states that, for a symbol $\varphi \in L^\infty$, $\varphi \equiv \bar{f} + g \ \ (f, g \in H^2)$, the Toeplitz operator T_φ acting on the Hardy space of the unit circle is hyponormal if and only if $f = c + T_{\bar{h}} g$, for some $c \in \mathbb{C}$, $h \in H^\infty$, $\|h\|_\infty \leq 1$. In this talk we consider possible versions of this result in the Bergman space case. Concretely, we consider Toeplitz operators on the Bergman space of the unit disk, with symbols of the form

$$\varphi \equiv \alpha z^n + \beta z^m + \gamma \bar{z}^p + \delta \bar{z}^q,$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ and $m, n, p, q \in \mathbb{Z}_+$, $m < n$ and $p < q$. By studying the asymptotic behavior of the action of T_φ on a particular sequence of vectors, we obtain a sharp inequality involving the above mentioned data. This inequality improves a number of existing results, and it is intended to be a precursor of basic necessary conditions for joint hyponormality of tuples of Toeplitz operators acting on Bergman spaces in one or several complex variables.

DMITRY KHAVINSON, University of South Florida
Vanishing of reproducing kernels in spaces of analytic functions

In most situations we are accustomed to , e.g., Bergman and Hardy spaces in the disk, the reproducing kernels do not vanish. Neither they do if we consider the later spaces with fairly general weights, for example comprised from moduluses of analytic functions. Yet in the "cut-off spaces" formed by polynomials of degree less or equal to n this is not necessarily true. We shall discuss what is known and the numerous compelling open problems that remain.

TRIEU LE, University of Toledo
Commutants of Separately Radial Toeplitz Operators on the Bergman Space

If φ is a bounded separately radial function on the unit ball, the Toeplitz operator T_φ is diagonalizable with respect to the standard orthogonal basis of monomials on the Bergman space. Given such a function φ, we characterize bounded functions ψ for which T_ψ commutes with T_φ. Several examples will be discussed to illustrate our results.

MARIBEL LOAIZA, CINVESTAV, Mexico
On Toeplitz operators on the poly harmonic Bergman space

Consider the upper half plane Π with the Lebesgue measure. Although the harmonic Bergman space $b^2(\Pi)$ is represented in terms of the Bergman and the anti-Bergman spaces, Toeplitz operators acting on $b^2(\Pi)$ behave different from those acting on the Bergman space. For example, contrary to the case of the Bergman space, the C*-algebra generated by Toeplitz operators with homogeneous symbols acting on the harmonic Bergman space is not commutative. On the other hand, the harmonic Bergman space is contained in each poly harmonic Bergman space, thus, it is natural to study Toeplitz operators acting on the last spaces. With this in mind, in this talk we study the C*-algebra generated by Toeplitz operators with homogeneous symbols acting on the poly harmonic Bergman space of the upper half plane.

JAVAD MASHREGHI, Laval University
The Gleason–Kahane–Zelazko theorem for modules

Let $T : H^p \to H^p$ be a linear mapping (no continuity assumption). What can we say about T if we assume that "it preserves outer functions"? Another related question is to consider linear functionals $T : H^p \to \mathbb{C}$ (again, no continuity assumption) and ask about those functionals whose kernels do not include any outer function. We study such questions via an abstract result which can be interpreted as the generalized Gleason–Kahane–Żelazko theorem for modules. In particular, we see that continuity of endomorphisms and functionals is a part of the conclusion.

This is a joint work with T. Ransford.

RAUL QUIROGA-BARRANCO, Cimat, Mexico
Toeplitz operators, special symbols and moment maps

Let us denote by $\mathbb{P}^n(\mathbb{C})$ the n-dimensional complex projective space. Our setup considers the weighted Bergman spaces over $\mathbb{P}^n(\mathbb{C})$ and their corresponding Toeplitz operators. Among the latter, we have special interest on those Toeplitz operators whose symbols are quasi-radial and quasi-homogeneous. Generally speaking, this means that, in a certain sense, the symbols depend only on the radial and spherical parts of subsets of the homogeneous coordinates. It turns out that such symbols, and thus their Toeplitz operators, can be related to the structure of toric manifold on $\mathbb{P}^n(\mathbb{C})$. We will describe such topological and geometric relationships.

This is joint work with M. A. Morales-Ramos and A. Sanchez-Nungaray.

THOMAS RANSFORD, Université Laval
Cyclicity in the harmonic Dirichlet space

The harmonic Dirichlet space $\mathcal{D}(\mathbb{T})$ is the Hilbert space of functions $f \in L^2(\mathbb{T})$ such that

$$\|f\|^2_{\mathcal{D}(\mathbb{T})} := \sum_{n\in\mathbb{Z}} (1+|n|)|\hat{f}(n)|^2 < \infty.$$

We give sufficient conditions for f to be cyclic in $\mathcal{D}(\mathbb{T})$, in other words, for $\{\zeta^n f(\zeta) : n \geq 0\}$ to span a dense subspace of $\mathcal{D}(\mathbb{T})$. (Joint work with E. Abakumov, O. El-Fallah and K. Kellay.)

ARMANDO SANCHEZ-NUNGARAY, Universidad Veracruzana
Commutative algebras of Toeplitz operators on the Siegel domain

We describe several ways of how the symbols, subordinated to the nilpotent group of biholomorphisms of the unit ball (i.e., invariant under the action of a subgroup of the nilpotent group), generate Banach (and even C^*) algebras that are commutative on each weighted Bergman space.

Recall for completeness that the nilpotent group of biholomorphisms of the Siegel domain D_n, the unbounded realization of the unit ball in $\mathbb{C}^n$, is isomorphic to $\mathbb{R}^{n-1} \times \mathbb{R}_+$ with the following group action

$$(b,h) : (z', z_n) \in D_n \longmapsto (z' + b, z_n + h + 2iz' \cdot b + i|b|^2) \in D_n,$$

for each $(b,h) \in \mathbb{R}^{n-1} \times \mathbb{R}_+$.

The key role in our study is played by the direct integral decomposition of the isomorphic image of the Bergman space on the Siegel domain, which is direct integral where each component is a weighted Fock spaces. We describe the action of Toeplitz operators with certain symbols as a direct integral of scalar multiplication operators and a direct integral of Toeplitz operators with the same symbol on the weighted Fock spaces. Note that all the above symbols are invariant under the action of the subgroup $\mathbb{R}^\ell \times \mathbb{R}_+$ ($\ell < n-1$) of the nilpotent group.

MICHAEL STESSIN, SUNY at Albany
Spectral characterization of representations of symmetry groups

Joint work with Z.Cuckovic.

Joint projective spectra of operator tuples generalize projective determinantal hypersurfaces that have been studied since 1920s. It was shown in a recent paper of Stessin and Tchernev that the appearance of certain quadrics in two-dimensional sections of joint spectrum of a tuple of self-adjoint operators implies the existence of subspace invariant for the tuples such that the group generated by restrictions of the operators to this subspace represents a Coxeter group.

We further investigate the connection between determinantal hypersurfaces and representations of Symmetry groups. Our main result is stated as follows.

Theorem. Let G be one of the groups A_n, B_n or a dihedral group $Ip(n)$ and let ρ_1 and ρ_2 be two representations of G. If the join projective spectra of images of Coxeter generators of G under ρ_1 and ρ_2 are the same,

$$\sigma(\rho_1(w_1), ..., \rho_1(w_n), I) = \sigma(\rho_2(w_1), ..., \rho_2(w_n), I),$$

then ρ_1 and ρ_2 are equivalent.

NIKOLAI VASILEVSKI, CINVESTAV, Mexico City
Toeplitz operators defined by sesquilinear forms

The classical theory of Toeplitz operators in spaces of analytic functions (Hardy, Bergman, Fock, etc spaces) deals usually with symbols that are bounded measurable functions on the domain in question. A further extension of the theory was made for symbols being unbounded functions, measures, and compactly supported distributions.

For reproducing kernel Hilbert spaces we describe a certain common pattern, based on the language of sesquilinear forms, that permits us to introduce a further substantial extension of a class of admissible symbols that generate bounded Toeplitz operators. Although the approach is unified for all reproducing kernel Hilbert spaces, for concrete operator consideration in this talk we restrict ourselves to Toeplitz operators acting on the standard Fock and Bergman spaces, as well as, on the Herglotz space of solutions of the Helmholtz equation.

The talk is based on a joint work with Grigori Rozenblum, Chalmers University of Technology, Gothenburg, Sweden.

YUNUS ZEYTUNCU, University of Michigan-Dearborn

Compactness of Hankel and Toeplitz operators on domains in $\mathbb{C}^n$

In this talk, I will present various characterizations of compactness of some canonical operators on domains in $\mathbb{C}^n$. I will highlight how complex geometry of the boundary of the domain plays a role in these characterizations. In particular, I will prove that on smooth bounded pseudoconvex Hartogs domains in $\mathbb{C}^2$ compactness of the $\overline{\partial}$-Neumann operator is equivalent to compactness of all Hankel operators with symbols smooth on the closure of the domain. The talk is based on recent joint projects with Željko Čučković and Sönmez Şahutoğlu.

RUHAN ZHAO, SUNY Brockport

Closures of Hardy and Hardy-Sobolev spaces in the Bloch type space on the unit ball

For $0 < \alpha < \infty$, $0 < p < \infty$ and $0 < s < \infty$, we characterize the closures in the α-Bloch norm of α-Bloch functions that are in a Hardy space H^p and in a Hardy-Sobolev space H^p_s on the unit ball of $\mathbb{C}^n$. This is a joint work with Jasbir Singh Manhas.

NINA ZORBOSKA, University of Manitoba

Intrinsic operators on spaces of holomorphic functions

I will talk about the boundedness and compactness of a large class of operators, mapping from general Banach spaces of holomorphic functions into the so called growth spaces. This class of operators contains some widely studied specific operators such as, for example, the composition, the multiplication, and the integral operators. I will present few results which generalize the previously known specific cases, and which show that the boundedness and compactness of the class of intrinsic operators depends largely on the behaviour over the point evaluation functions.