
Optimization and Control

Optimisation et contrôle

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LUIS BRICEÑO ARIAS, Universidad Técnica Federico Santa María
Forward-Backward-Half forward splitting for solving monotone inclusions

Tseng's algorithm finds a zero of the sum of a maximally monotone operator and a monotone- Lipschitz operator by evaluating the latter twice per iteration. In this paper, we modify Tseng's algorithm for finding a zero of the sum of three operators, where we add a cocoercive operator to the inclusion. Since the sum of a cocoercive and a monotone-Lipschitz operator is monotone and Lipschitz, we could use Tseng's method for solving this problem, but implementing both operators twice per iteration and without taking into advantage the cocoercivity property of one operator. Instead, in our approach, although the Lipschitz operator must still be evaluated twice, we exploit the cocoercivity of one operator by evaluating it only once per iteration. Moreover, when the cocoercive or monotone-Lipschitz operators are zero it reduces to Tseng's or forward-backward splittings, respectively, unifying in this way both algorithms. In addition, we provide a variable metric version of the proposed method but including asymmetric linear operators in the computation of resolvents and the single-valued operators involved. This approach allows us to extend previous variable metric versions of Tseng's and forward-backward methods and simplify their conditions on the underlying metrics. We also exploit the case when the asymmetric linear operator is triangular by blocks in the primal-dual product space for solving primal-dual composite monotone inclusions, obtaining Gauss- Seidel type algorithms which generalize several primal-dual methods available in the literature.

MARÍA SOLEDAD ARONNA, Escola de Matemática Aplicada, FGV
Second order analysis of bilinear optimal control

We present second order optimality conditions for a bilinear optimal control problem governed by a strongly continuous semigroup operator, the control entering linearly in the cost function. We derive first and second order optimality conditions in this quite general framework. We then apply the results to the heat and wave equations. We comment on the extension to the complex setting and application to the Schrodinger equation.

MARIO BRAVO, Universidad de Santiago de Chile
Sharp convergence rates for averaged non expansive maps

We consider the well-known Krasnoselskii-Mann (KM) sequential averaging iteration

$$x_{n+1} = (1 - \alpha_{n+1}) x_n + \alpha_{n+1} T x_n, \quad (\text{KM})$$

where $T : C \rightarrow C$ is nonexpansive with C bounded closed convex in a Banach space, and $0 \leq \alpha_n \leq 1$. This provides a method to compute fixed points and to establish their existence, and it finds many applications in Optimization and beyond.

A key step in proving the convergence of the iteration is to show that $\|x_n - T x_n\| \rightarrow 0$ as $n \rightarrow \infty$. This property is named *asymptotic regularity* and has been studied under different assumptions on the space and the set C . In 1976, Browder and Petryshyn showed that asymptotic regularity holds in every Banach space, provided $\sum_i \alpha_i = +\infty$. In 1992, Baillon and Bruck conjectured a bound that implies all the previous results: there exists a universal constant κ such that

$$\|x_n - T x_n\| \leq \kappa \frac{\text{diam}(C)}{\sqrt{\sum_{i=1}^n \alpha_i (1 - \alpha_i)}}. \quad (\text{B})$$

Cominetti et al. (2014) proved that (B) holds with $\kappa = 1/\sqrt{\pi}$. In this talk we show that this constant is sharp. Our approach builds on the idea of constructing a sequence of bounds $\|x_m - x_n\| \leq d_{mn}$ for $m \leq n$, which are valid for every nonexpansive

map. The d_{mn} 's are defined by a recursive sequence of discrete optimal transport problems, and can be interpreted as absorption probabilities of a suitable Markov process.

We also discuss how to extend this approach to establish new rates of convergence for inexact versions of the (KM) iteration. Based on joint work with Roberto Cominetti and Matias Pavez-Signe.

ORESTES BUENO, Universidad del Pacífico
The pseudomonotone polar for multivalued operators

In this talk, we study the pseudomonotonicity of multivalued operators from the point of view of polarity, in an analogous way as the well-known monotone polar due to Martínez-Legaz and Svaiter. We show that this new polar, adapted for pseudomonotonicity, possesses analogous properties to the monotone and quasimonotone polar, among which are a characterization of pseudomonotonicity, maximality and pre-maximality. Furthermore, we characterize the notion of D -maximal pseudomonotonicity introduced by Hadjisavvas.

RAFAEL CORREA, Universidad de Chile
On Brøndsted-Rockafellar's Theorem for convex lower semicontinuous epi-pointed functions in locally convex spaces

It is known that the Brøndsted-Rockafellar Theorem is not valid outside Banach spaces (see [1]). The motivation for this work is to answer the question of whether Brøndsted-Rockafellar is valid for convex functions defined on a locally convex space. The response we got is that it is valid for the class of functions called epi-pointed. Also we will see that the proof is direct in the sense that it is not based on Ekeland's or Bishop-Phelps' variational principle as the usual demonstrations. Furthermore, this result has as an immediate corollary the Brøndsted-Rockafellar Theorem in the setting of reflexive Banach spaces. Finally, we obtain Brøndsted-Rockafellar's Theorem, directly using our result in the framework of general Banach spaces.

References

- [1] A. Brøndsted and R. T. Rockafellar. On the subdifferentiability of convex functions. *Proc. Amer. Math. Soc.*, 16 (1965) 605-611.

JOHN COTRINA, Universidad del Pacífico
Remarks on p -cyclic monotone linear operators

The goal of this talk is to study the p -cyclic monotone linear operators. More precisely, we give a new characterization and provide several new properties.

VALERIANO ANTUNES DE OLIVEIRA, UNESP - Univ. Estadual Paulista, C.S.J. Rio Preto, S. Paulo, Brasil
The constant rank constraint qualification in the continuous-time context

A constant rank type constraint qualification is defined for continuous-time nonlinear programming problems with equality constraints. Necessary optimality conditions of Karush-Kuhn-Tucker type are obtained under this constraint qualification. It is worthy mentioning that a constant rank condition is less restrictive than the full rank one, which is commonly assumed in Karush-Kuhn-Tucker type theorems.

Joint work with Moisés Rodrigues Cirilo do Monte.

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YBOON GARCÍA, Universidad del Pacífico

Existence results for equilibrium problem

In this talk, we introduce the notion of regularization of bifunctions in a similar way as the well known convex, quasiconvex and lower semicontinuous regularizations due to Crouzeix. We show that the Equilibrium Problems associated to bifunctions and their regularizations are equivalent in the sense of having the same solution set. Also, we present new existence results of solutions for Equilibrium Problems.

CRISTOPHER HERMOSILLA, Universidad Técnica Federico Santa María

Constrained and impulsive Linear Quadratic control problems

The aim of this talk is to show some results concerning the value function of a time-continuous linear quadratic optimal control problem with input and state constraints. No coercive assumptions are made, which leads to optimal control problems whose trajectories are of bounded variation rather than merely absolutely continuous. Our approach is based on classical convex analysis, and we establish a Legendre-Fenchel type equality between the value function of the linear quadratic problem and its dual problem. Furthermore, we present a characteristic method for describing the Hamiltonian evolution of the subgradients of the value function.

ALEJANDRO JOFRÉ, Universidad de Chile, CMM and DIM

Variance-based stochastic extragradient methods with linear search for stochastic variational inequalities

We propose stochastic extragradient methods for stochastic variational inequalities with a linear search requiring only pseudo-monotonicity of the operator and no knowledge of the Lipschitz constant L . We provide convergence and complexity analysis, allowing for an unbounded feasible set, unbounded operator, non-uniform variance of the oracle and we do not require any regularization. We also prove the generated sequence is bounded in L^p . Alongside the stochastic approximation procedure, we iteratively reduce the variance of the stochastic error. Our methods cope with stepsizes bounded away from zero and attain the near-optimal oracle complexity $O(\log_{1/\theta} L) \cdot \epsilon^{-2} \cdot [\ln(\epsilon^{-1})]^{1+b}$ and an accelerated rate $O(1/K)$ in terms of the mean (quadratic) natural residual and the mean D-gap function, where K is the number of iterations required for a given tolerance $\epsilon > 0$ for arbitrary $\theta \in (0, 1)$ and $b > 0$. Explicit estimates for the convergence rate, oracle complexity and the p -moments are given depending on problem parameters and the distance of initial iterates to the solution set.

MICHELE PALLADINO, Penn State University

Growth Model for Tree Stems and Vines

In this talk, we propose a model describing the growth of tree stems and vine, taking into account also the presence of external obstacles. The system evolution is described by an integral differential equation which becomes discontinuous when the stem hits the obstacle. The stem feels the obstacle reaction not just at the tip, but along the whole stem. This fact represents one of the main challenges to overcome, since it produces a cone of possible reactions which is not normal with respect to the obstacle. However, using the geometric structure of the problem and optimal control tools, we are able to prove existence and uniqueness of the solution under natural assumptions on the initial data.

PEDRO PÉREZ-AROS, Universidad de Chile

Subdifferential characterization of probability functions under Gaussian distribution

This work provides formulae for the subdifferential of the probability function

$$\varphi(x) = \mathbb{P}(g(x, \xi) \leq 0),$$

where $(\Omega, \Sigma, \mathbb{P})$ is a probability space, ξ is an m -dimensional gaussian random vector, $g : X \times \mathbb{R}^m \rightarrow \mathbb{R}$ is locally Lipschitz and convex in the second variable and X is a separable reflexive Banach space. Applications for this class of functions can be found

in water management, telecommunications, electricity network expansion, mineral blending, chemical engineering, etc, where the constraint $\mathbb{P}(g(x, \xi) \leq 0) \geq p$ expresses that a decision vector x is feasible if and only if the random inequality $g(x, \xi) \leq 0$ is satisfied with probability at least p . The aim of this work is to extend the previous results (see [1, 2]) to infinite dimensional spaces X , using a weaker growth condition and assuming local Lipschitz continuity of g only, even when the probabilistic function φ could be non-Lipschitz.

References

- [1] W. van Ackooij and R. Henrion . Gradient formulae for nonlinear probabilistic constraints with Gaussian and Gaussian-like distributions. *SIAM J. Optim.*, 24 (2014), no. 4, 1864-1889.
- [2] W. van Ackooij and R. Henrion . (Sub-) gradient formulae for probability functions of random inequality systems under Gaussian distribution. *SIAM/ASA J. Uncertain. Quantif.*, 5 (2017), no. 1, 63-87.

HÉCTOR RAMÍREZ, Universidad de Chile

BIOREMEDIATION OF WATER RESOURCES: AN OPTIMAL CONTROL APPROACH

This talk deals with the bioremediation, in minimal time, of a water resource (such as lakes, reservoirs, etc.) using a single continuous bioreactor. The bioreactor is connected to the reservoir through several pumps. Typically, one pump extracts polluted water and other one injects back sufficiently clean water with the same flow rate. However, we also analyze more complex pumps configurations. So, we state minimal-time optimal control problems where the control variables are related to the inflow rates of the pumps. For those problems, we analyze the existence of their solutions as well as their optimal synthesis (via Pontryaguin's Principle). We also obtain, for some configurations, explicit bounds on their value functions via Hamilton–Jacobi–Bellman techniques.

FRANCISCO SILVA, XLIM-Université de Limoges

On the variational formulation of some stationary second order MFGs

In this talk we consider an extension of the work by A. Mészáros and myself (15') dealing with variational stationary MFGs with density constraints. We consider general Hamiltonians, satisfying a suitable growth condition, and also rather general coupling terms. For systems with and without density constraint we establish the existence of solutions using a variational technique and we also improve some of our previous results. Numerical applications of this approach will also be discussed.

GERALDO NUNES SILVA, Universidade Estadual Paulista - UNESP

Minmax control problems and the Hamilton-Jacobi Equation

We look at minmax optimal control problems where both the cost and the associated dynamics are dependent on parameters that are contained in a nonempty compact metric space \mathcal{A} . The optimization is taken over the worst case scenario with the parameters varying in the set \mathcal{A} . We provide optimality conditions via Hamilton-Jacobi theory. The methodology employed is by first providing the optimality conditions for problems with finite sets \mathcal{A} and then approximating the infinite abstract set by increasingly finite sets and proving the convergence of the optimality conditions derived for problems posed with finite sets to general minmax problems with abstract compact metric spaces \mathcal{A} .