Number Theory & Analysis
Théorie des nombres et analyse

(Org: **Harald Andres Helfgott** (Universitat Gottingen), **Roberto Miatello** (Universidad Nacional de Cordoba) and/et **Maksym Radziwill** (McGill University))

CLARA ALDANA, Universite du Luxembourg

Determinants of Laplacians on surfaces with singularities

I will talk about certain aspects of determinants of Laplace operators on surfaces. I will consider two settings: surfaces with cusps and funnels, and surfaces with conical singularities.

I will mention some of the results that we obtained for determinants of Laplacians in these two cases, the technical difficulties that appeared there and how we solved them. The results for surfaces with cusps and funnels are joint work with Pierre Albin and Frederic Rochon, and the part about surfaces with conical singularities are joint work with Julie Rowlett.

MISHA BELOLIPETSKY, IMPA

Lehmer's problem and triangulations of arithmetic hyperbolic 3-orbifolds

A triangulation of a hyperbolic orbifold is called good if all the simplices are geodesic and l-dimensional skeleton of the singular set is contained in the l-skeleton of the triangulation for every l. The purpose of the talk is show how the known quantitative results towards Lehmer's problem on the Mahler measure of non-cyclotomic polynomials can be applied to produce good triangulations of arithmetic hyperbolic 3-orbifolds with small number of simplices. More precisely, we show that for any $\epsilon > 0$, there is a constant $V_0 = V_0(\epsilon)$ such that any closed orientable arithmetic hyperbolic 3-orbifold of volume $V_{hyp} \geq V_0$ has a good triangulation with at most $V_{hyp}^{1+\epsilon}$ simplices and vertex degree bounded above by an absolute constant.

EMANUEL CARNEIRO, IMPA - Rio de Janeiro

Bandlimited approximations and estimates for the Riemann zeta-function

We provide explicit upper and lower bounds for the argument of the Riemann zeta-function and its antiderivatives on the critical line and in the critical strip, under the assumption of the Riemann hypothesis. Our tools come not only from number theory, but also from Fourier analysis and approximation theory. An important element in our strategy is the ability to solve a Fourier optimization problem with constraints, namely, the problem of majorizing certain real-valued even functions by bandlimited functions, optimizing the $L^1(\mathbb{R})-$error. Deriving explicit formulae for the Fourier transforms of such optimal approximations plays a crucial role in our approach. The most recent works are joint with A. Chirre and M. Milinovich.

JOHN FRIEDLANDER, University of Toronto

On Dirichlet L-functions

We discuss some relations among their values at $s = 1$, their character values at primes, and their zero-free regions. (joint with H. Iwaniec)

ALIREZA GOLSEFIDY, University of California, San Diego

Super-approximation

Suppose G is a finitely generated linear group over a global field. Super-approximation results imply that, under some conditions on the Zariski-closure of G, the Cayley graphs of (certain) finite congruence quotients of G are expanders, that means roughly highly connected. In this talk I will present some of the best known super-approximation results. If time permits, some of the applications of super-approximation will be mentioned.

HENRYK IWANIEC, Rutgers University
Critical zeros of L-functions

I will discuss various issues related with the zeros on the critical line of families of L-functions. These include: the Riemann zeta function, the Dirichlet L-functions, the Hecke L-functions for quadratic number fields and the L-functions of elliptic curves.

HENRY KIM, University of Toronto
The least prime in a conjugacy class

Let K be a number field with the discriminant d_K with its Galois closure $\hat{K}$. Let C be a conjugacy class of $Gal(\hat{K}/\mathbb{Q})$. Let $n_{K,C}$ be the least prime p which is ramified or whose Frobenius automorphism Frob_p does not belong to C. Then under GRH, $n_{K,C}$ is $O((\log|d_K|)^2)$. We prove two unconditional results regarding $n_{K,C}$. First, the average of $n_{K,C}$ in a family of S_n-fields ($n = 3, 4, 5$) is a constant. Second, in a family of S_n-fields ($n = 3, 4, 5$), except for a density zero set, $n_{K,C} = O(\log|d_K|)$.

ALEX KONTOROVICH, Rutgers
Beyond Expansion and Arithmetic Chaos

We will describe recent progress in our ongoing program with Jean Bourgain to understand a number of different problems through the lens of thin orbits. An important role will be played by the production of levels of distribution (in certain Affine Sieves) which go "beyond expansion."

EMILIO LAURET, Universidad Nacional de Córdoba
One-norm spectrum of a lattice

In 1964, John Milnor gave the first example of isospectral non-isometric compact Riemannian spaces. To do this, he related the spectrum of the Laplace operator on a torus, and the (Euclidean) norm of the vectors of the (corresponding) dual lattice. Consequently, a pair of lattices with the same theta function induces a pair of isospectral tori.

In this talk, we will introduce a new relation between the spectrum of a lens space (a sphere over a cyclic group), and the one-norm (sum of the absolute values of the entries) of the vectors in an associated lattice. We associate to each lattice, the one-norm generating function defined as follows: the power series whose k-th term is the number of vectors in the lattice with one-norm equal to k.

We will show that two lens spaces are isospectral if and only if their corresponding lattices have the same one-norm generating function. Furthermore, we will prove that the generating function is a rational function, and consequently, a finite part of the spectrum determines the whole spectrum.

This is a joint work with Roberto Miatello and Juan Pablo Rossetti.

AMIR MOHAMMADI, University of California, San Diego
Effective equidistribution of certain adelic periods

We will present a quantitative equidistribution result for adelic homogeneous subsets whose stabilizer is maximal and semisimple. Some number theoretic applications will also be discussed. This is based on a joint work with Einsiedler, Margulis and Venkatesh.

CORENTIN PERRET-GENTIL, Centre de Recherches Mathématiques
Quotients of elliptic curves over finite fields

In fixed characteristic $p > 0$, there are only finitely many supersingular elliptic curves. Given a finite subgroup of such a curve, we can form the quotient, which is still a supersingular elliptic curve, isogenous to the base curve. For a family of subgroups of growing size (for example all cyclic subgroups of given cardinality), we would like to know how these quotients distribute in the isomorphism classes of supersingular elliptic curves. This question is related to the study of the security of recent cryptographic schemes using isogenies. The techniques involve applying the Riemann hypothesis over finite fields (in a general version) to exponential sums having high degree or to Jacobians of modular curves. Similar questions can be addressed for ordinary curves.

KANNAN SOUNDARARAJAN, Stanford University
Value distribution of L-functions.

I will discuss some work with Radziwill motivated by the Keating-Snaith conjectures that central values of L-functions should be log normal.

LOLA THOMPSON, Oberlin College
Bounded gaps between primes and the length spectra of arithmetic hyperbolic 3-orbifolds

In 1992, Reid posed the question of whether hyperbolic 3-manifolds with the same geodesic length spectra are necessarily commensurable. While this is known to be true for arithmetic hyperbolic 3-manifolds, the non-arithmetic case is still open. Building towards a negative answer to Reid's question, Futer and Millichap have recently constructed infinitely many pairs of non-commensurable, non-arithmetic hyperbolic 3-manifolds which have the same volume and whose length spectra begin with the same first n geodesic lengths. In the present talk, we show that this phenomenon is surprisingly common in the arithmetic setting. In particular, given any arithmetic hyperbolic 3-orbifold derived from a quaternion algebra and any finite subset S of its geodesic length spectrum, we produce, for any $k \geq 2$, infinitely many k-tuples of arithmetic hyperbolic 3-orbifolds which are pairwise non-commensurable, have geodesic length spectra containing S, and have volumes lying in an interval of (universally) bounded length. The main technical ingredient in our proof is a bounded gaps result for prime ideals in number fields lying in Chebotarev sets. This talk is based on joint work with B. Linowitz, D. B. McReynolds, and P. Pollack.

ANTHONY VARILLY-ALVARADO, Rice University
On a uniform boundedness conjecture for Brauer groups of K3 surfaces

Braurer groups of K3 surfaces behave in many ways like torsion points of elliptic curves. In 1996, Merel showed that torsion groups of elliptic curves are uniformly bounded across elliptic curves defined over number fields of fixed degree. I will discuss a conjecture pointing towards an analogous statement for K3 surfaces, and survey recent mounting evidence for it.