Nonlinear Partial Differential Equations
Équations aux dérivées partielles (ÉDP) non linéaires
(Org: **Manuel del Pino** (Universidad de Chile) and/et **Jérôme Vétois** (McGill University))

MONICA CLAPP, Universidad Nacional Autónoma de México
Towers of nodal bubbles for the Bahri-Coron problem in punctured domains

Let Ω be a bounded smooth domain in $\mathbb{R}^N$ which contains a ball of radius R centered at the origin, $N \geq 3$. Under suitable symmetry assumptions, for each $\delta \in (0, R)$, we establish the existence of a sequence $(u_{m,\delta})$ of nodal solutions to the critical problem

$$\begin{cases} -\Delta u = |u|^{2^*-2}u & \text{in } \Omega_\delta := \{x \in \Omega : |x| > \delta\}, \\ u = 0 & \text{on } \partial\Omega_\delta, \end{cases}$$

where $2^* := \frac{2N}{N-2}$ is the critical Sobolev exponent. We show that, if Ω is strictly starshaped, then, for each $m \in \mathbb{N}$, the solutions $u_{m,\delta}$ concentrate and blow up at 0, as $\delta \to 0$, and their limit profile is a tower of nodal bubbles, i.e., it is a sum of rescaled nonradial sign-changing solutions to the limit problem

$$\begin{cases} -\Delta u = |u|^{2^*-2}u, \\ u \in D^{1,2}(\mathbb{R}^N), \end{cases}$$

centered at the origin.

This is joint work with Jorge Faya (Universidad de Chile) and Filomena Pacella (Università "La Sapienza" di Roma).

JUAN DAVILA, Universidad de Chile
Vortices for the 2D Euler equation

In this talk we present an asymptotic expansion of solutions for the 2D Euler equation with highly concentrated vorticity around a finite number of points. Compared to previous results we obtain a finer description of the solution. We do this by exploiting a connection with the Liouville equation. This is joint work with Manuel del Pino (Universidad de Chile), Monica Musso (Universidad Catolica de Chile) and Juncheng Wei (University of British Columbia).

ROBERT JERRARD, University of Toronto
Dynamics of defects in a semilinear wave equation

It is known that a semilinear wave equation with a bistable nonlinearity exhibits, for suitable initial data, an interface whose evolution approximately sweeps out a timelike extremal surface in Minkowski space. We present a new approach to this issue, one that yields a much sharper description of the interface, allows for a wider class of nonlinearities, and that may extend to a range of related problems. This is joint work with Manuel del Pino and Monica Musso.

JOSE MAZON, Universitat de Valencia
Kurdyka-Łojasiewicz-Simon inequality for gradient flows in metric spaces

The classical Łojasiewicz inequality and its extensions by Simon and Kurdyka have been a considerable impact on the analysis of the large time behaviour of gradient flow in Hilbert spaces. Our aim is to adapt the classical Kurdyka-Łojasiewicz and Łojasiewicz-Simon inequalities to the general framework gradient flow in metric spaces. We show that the validity of a Kurdyka-Łojasiewicz inequality imply trend to equilibrium in the metric sense, and the Kurdyka-Łojasiewicz inequality has the advantage to derive decay estimates of the trend to equilibrium and finite time of extinction. Also we study the relation between Kurdyka-Łojasiewicz inequality and the existence of talweg. The entropy method have proved to be very useful to study the large time behaviour of solutions to many EDP's. This method is based in the entropy-entropy production/disipation (EEP) inequality, which correspond to Kurdyka-Łojasiewicz inequality, and also in the entropy transportation (ET) inequality. We show that for geodesically convex functionals Kurdyka-Łojasiewicz inequality and entropy transportation (ET) inequality are equivalent. We apply our general results to gradient flow in Banach spaces and in spaces of probability measures with Wasserstein distances. For the energy functional associated with a doubly-nonlinear equations on $\mathcal{R}^N$ we obtain the equivalence between Łojasiewicz-Simon inequality, generalized log-Sobolev inequality and p-Talagrand inequality; also we get decay estimates for its solutions. Finally we apply our results to metric spaces with Ricci curvature bounds from below, getting that, in this context, a p-Talagrand inequality is equivalent to a Łojasiewicz-Simon inequality.

Joint work with Daniel Hauer (Sydney University)

JULIO ROSSI, Buenos Aires University
Maximal operators for the p-Laplacian family

We prove existence and uniqueness of viscosity solutions for the following problem:

$$\max\{-\Delta_{p_1}u(x),\, -\Delta_{p_2}u(x)\} = f(x)$$

in a bounded smooth domain Ω with $u = g$ on $\partial\Omega$. Here $-\Delta_p u = (N+p)^{-1}|Du|^{2-p}\text{div}(|Du|^{p-2}Du)$ is the 1-homogeneous $p-$Laplacian and we assume that $2 \leq p_1, p_2 \leq \infty$. This equation appears naturally when one considers a tug-of-war game in which one of the players (the one who seeks to maximize the payoff) can choose at every step which are the parameters of the game that regulate the probability of playing a usual Tug-of-War game (without noise) or to play at random. Moreover, the operator $\max\{-\Delta_{p_1}u(x),\, -\Delta_{p_2}u(x)\}$ provides a natural analogous with respect to $p-$Laplacians to the Pucci maximal operator for uniformly elliptic operators.

We provide two different proofs of existence and uniqueness for this problem. The first one is based in pure PDE methods (in the framework of viscosity solutions) while the second one is more connected to probability and uses game theory.

Joint work with P. Blanc and J. Pinasco

BOYAN SIRAKOV, PUC-Rio
Exact multiplicity results for a nonlinear elliptic problem, and geometric structure of the set of solutions

We revisit a very classical result in the theory of nonlinear elliptic PDE - the Ambrosetti-Prodi problem. This problem is essentially solved when the underlying operator is self-adjoint, and a rather complete description of the set of solutions is available.

On the other hand, for non-divergence form elliptic operators only partial results are available. We develop a method based on elliptic regularity and maximum principle techniques, which lets us prove that the same results which were known in the divergence case are valid for non-divergence form operators, and even obtain some new results for self-adjoint operators. In particular, we show that the Ambrosetti-Prodi operator is a global fold from $W^{2,p}$ to L^p, $p \geq n$.

To our knowledge this is the first result of exact multiplicity of solutions (i.e. exact number of solutions different from 0 or 1) for non-divergence form elliptic PDE.

This is a joint work with C. Tomei and A. Zaccur.

YANNICK SIRE, Johns Hopkins University
A singular perturbation problem for the fractional Allen-Cahn equation

I will describe some convergence result for a singular perturbation of the fractional Allen-Chan equation involving powers of the laplacian less than $1/2$. In this case, one converges in a suitable sense to stationary minimal nonlocal surfaces that I will describe precisely. The convergence happens to be strong due to a deep result of Geometric Measure Theory due to Marstrand. This is a feature of the non locality of the problem.