Noncommutative Geometry and Quantization
Géométrie non commutative et quantification

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ALCIDES BUSS, UFSC

Groupoid actions - The symmetries of noncommutative spaces

Groupoids are powerful objects with many applications in the theory of Operator Algebras. Important examples of C^*-algebras, like rotation algebras and Cuntz algebras have canonical groupoid models, that is, they can be described in terms of a groupoid. Properties of the C^*-algebra can then be read from the underlying groupoid model.

Groupoids can also be used to describe symmetries of C^*-algebras and the aim of this talk will be to explain how this works. More specifically, we introduce a notion of "action" of one groupoid H on another groupoid G using the theory of groupoid fibrations and explain how this induces an "action" of H on the groupoid C^*-algebra $C^*(G)$. In this setting we have a transformation groupoid $G \rtimes H$ and its C*-algebra $C^*(G \rtimes H)$ can be described as a sort of "generalised crossed product" $C^*(G) \rtimes H$.

ALEJANDRO CABRERA, Universidade Federal do Rio de Janeiro

A geometric approach to some equivalent C-algebras*

A result of M.A. Rieffel from 1981 states that two C*-algebras associated to irrational rotations by a and b on the circle are strongly Morita equivalent iff a and b are in the same orbit under the natural GL(2,Z) action. We give an alternative proof of this fact based on analyzing the geometry of the underlying differentiable stacks given by the circle modulo the rotation actions. We also mention possible further research directions into studying the geometry of the quotient space by a general dynamics. This is joint work with M. del Hoyo and E. Pujals.

EUGENIA ELLIS, Universidad de la República

Algebraic quantum kk-theory

Let $\mathcal{G}$ be an algebraic quantum group in the sense of Van Daele. We define a bivariant K-theory on the category of $\mathcal{G}$-module algebras. For each pair (A, B) of $\mathcal{G}$-module algebras we define a group $\mathrm{kk}^{\mathcal{G}}(A, B)$ and consider the category $\mathfrak{KK}^{\mathcal{G}}$ whose objects are the $\mathcal{G}$-module algebras and the morphisms from A to B are the elements of $\mathrm{kk}^{\mathcal{G}}(A, B)$.

Consider the functor $j^{\mathcal{G}} : \mathcal{G}\text{-Alg} \to \mathfrak{KK}^{\mathcal{G}}$ which at the level of objects is the identity and at the level of morphisms sends $f : A \to B$ to its class $[f]$ in $\mathrm{kk}^{\mathcal{G}}(A, B)$. The category $\mathfrak{KK}^{\mathcal{G}}$ is triangulated and $j^{\mathcal{G}}$ is an excisive, homotopy invariant and $\mathcal{G}$-stable functor. Moreover, it is the universal functor for these properties.

The Green-Julg Theorem in Kasparov KK-theory states that if G is a compact group and B is a G-C^*-algebra, then there exists an isomorphism

$$\mu : \mathrm{KK}^G(\mathbb{C}, B) \to \mathrm{KK}(\mathbb{C}, B \rtimes G).$$

The main theorem of this talk is a version of Green-Julg Theorem for $\mathfrak{KK}^{\mathcal{G}}$ when $\mathcal{G}$ is a semisimple Hopf algebra. Let A be an algebra and B a $\mathcal{G}$-module algebra then there exists an isomorphism

$$\psi : \mathrm{kk}^{\mathcal{G}}(A^\tau, B) \to \mathrm{kk}(A, B\#\mathcal{G})$$

where $B\#\mathcal{G}$ denotes the smash product. In particular, we obtain that $\mathrm{kk}^{\mathcal{G}}(\mathbb{C}, B) \simeq \mathrm{KH}(B\#\mathcal{G})$.

CARLA FARSI, University of Colorado

Semibranching function systems, representations, wavelets, and spectral triples for k-graphs

In joint work with Gillaspy, Kang, and Packer, we generalized the definition of semibranching function systems from directed graphs to finite higher-rank graphs (k-graphs). This enabled us to construct a wavelet-type orthogonal decomposition on the infinite path space of the k-graph. In subsequent joint work with Gillaspy, Julien, Kang, and Packer, we show that this wavelet decomposition is closely tied to the Cantor set spectral triples introduced by Pearson and Bellissard. In particular, we show that the Farsi-Gillaspy-Kang-Packer wavelet decomposition agrees with the decomposition as eigenspaces of the Laplace-Beltrami operators of the Pearson-Bellissard spectral triples. To do this, we recast the Cantor set spectral triples in the k-graph set-up by using the infinite path space of the k-graph as our Cantor set. Moreover, in joint work in progress with Gillaspy, Jorgensen, Kang, and Packer, we also study monic, atomic, and permutative representations for finite k-graphs associated to k-graph semibranching function systems, thus generalizing results on representations of Cuntz algebras to the k-graphs set-up.

FARZAD FATHIZADEH, California Institute of Technology (Caltech)
The term a_4 in the heat kernel expansion of noncommutative tori

The analog of the Riemann curvature tensor for noncommutative tori manifests itself in the term a_4 appearing in the heat kernel expansion of the Laplacian of curved metrics. This talk presents a joint work with Alain Connes, in which we obtain an explicit formula for the a_4 associated with a general metric in the canonical conformal structure on noncommutative two-tori. Our final formula has a complicated dependence on the modular automorphism of the state or volume form of the metric, namely in terms of several variable functions with lengthy expressions. We verify the accuracy of the functions by checking that they satisfy a family of conceptually predicted functional relations. By studying the latter abstractly we find a partial differential system which involves a natural flow and action of cyclic groups of order two, three and four, and we discover symmetries of the calculated expressions with respect to the action of these groups. At the end, I will illustrate the application of our results to certain noncommutative four-tori equipped with non-conformally flat metrics and higher dimensional modular structures.

ELIZABETH GILLASPY, WWU-Münster and University of Montana
Generalized gauge actions, KMS states, and Hausdorff dimension for higher-rank graphs

Inspired by work of McNamara, Exel-Laca, and Ionescu-Kumjian, we study generalized gauge actions for strongly connected higher-rank graphs (k-graphs). In our setting the generalized gauge action arises from a weight functor on the k-graph Λ combined with a real parameter β. We show that the same data also gives rise to a metric on the infinite path space Λ^∞ of our k-graph, and that the Hausdorff measure of the associated metric space is intimately related to the KMS states for the original generalized gauge action. This is joint work in progress with Carla Farsi, Sooran Kang, Nadia Larsen, and Judy Packer.

SHERRY GONG, MIT
Traces on reduced group C^*-algebras

In this talk I will be talking about traces on the reduced C^*-algebras of groups. In particular, we will look at when traces can detect certain elements in the K-theory of groups with a certain rapid decay property. We will also look in particular at hyperbolic groups and classify the traces on their reduced C^*-algebras.

MASOUD KHALKHALI, University of Western Ontario
Ricci Curvature in Noncommutative Geometry

Motivated by Gilkey's local formulae for asymptotic expansion of heat kernels in spectral geometry, we propose a definition of Ricci curvature in noncommutative settings. The Ricci operator of an oriented closed Riemannian manifold can be realized as a spectral functional, namely the functional defined by the zeta function of the full Laplacian of the de Rham complex, localized by smooth endomorphisms of the cotangent bundle and their trace. We use this formulation to introduce the Ricci functional in a noncommutative setting and in particular for curved noncommutative tori. This Ricci functional uniquely determines a density element, called the Ricci density, which plays the role of the Ricci operator. The main result of this paper provides an

explicit computation of the Ricci density when the conformally flat geometry of the noncommutative two torus is encoded by the modular de Rham spectral triple. Joint work with R. Floricel and A. Ghorbanpour.

SEVERINO TOSCANO DO REGO MELO, Universidade de São Paulo
K-theory of pseudodifferential operators with semiperiodic symbols on a cylinder.

Let B be a compact Riemannian manifold, let Ω denote the cylinder $\mathbb{R} \times B$, Δ_Ω its Laplace operator and $\Lambda = (1 - \Delta_\Omega)^{-1/2}$. Let $\mathfrak{A}$ denote the C*-algebra of bounded operators on $L^2(\mathbb{R} \times B)$ generated by all the classical pseudodifferential operators on $\mathbb{R} \times B$ of the form $L\Lambda^N$, N a nonnegative integer and L an N-th order differential operator whose (local) coefficients approach 2π-periodic functions at $+\infty$ and $-\infty$. Let $\mathfrak{E}$ denote the kernel of the continuous extension of the principal symbol to $\mathfrak{A}$. The problem of computing the K-theory index map $\delta_1(K_1(\mathfrak{A}/\mathfrak{E})) \to K_0(\mathfrak{E}) \simeq \mathbb{Z}^2$ on an element of $K_1(\mathfrak{A}/\mathfrak{E})$ is reduced to the problem of computing the Fredholm indices of two elliptic operators on the compact manifold $S^1 \times B$. In the case $B = S^1$, it follows from considerations about various exact sequences of C*-subalgebras of $\mathfrak{A}$ that δ_1 is onto and that $K_0(\mathfrak{A}) \simeq \mathbb{Z}^5$ and $K_1(\mathfrak{A}) \simeq \mathbb{Z}^4$. This talk is based on joint work with Patricia Hess.

JOSEPH MIGLER, The Ohio State University
Determinants of almost commuting operators

Determinant-type invariants known as torsion may be associated to collections of operators that commute modulo trace ideals. We will review the construction of these invariants and discuss recent work on their applications.

RUDY RODSPHON, Vanderbilt University
Quantizations and index theory

One way to describe succinctly local index theory on closed spin manifolds could be the following slogan of Quillen : Dirac operators are a "quantization" of connections, and index theory is a "quantization" of the Chern character. For non necessarily spin manifolds, pseudodifferential operators and their symbolic calculus play a crucial role in the original proofs of the index theorem. However, symbols may also be viewed as a deformation quantization of functions on the cotangent bundle, which has led to other fruitful approaches to index theory through a "quantization" process. Methods used in these two different quantization pictures do not seem to be quite related a priori. The upshot of the talk will be to see that these different theories might have more to tell to each other, and that far reaching index problems may be solved very directly from such an interaction.

XIANG TANG, Washington University in St. Louis
Roe C-algebra for groupoids and generalized Lichnerowicz Vanishing theorem*

We introduce the concept of Roe C*-algebra for a locally compact groupoid whose unit space is in general not compact, and that is equipped with an appropriate coarse structure and Haar system. Using Connes' tangent groupoid method, we introduce an analytic index for an elliptic differential operator on a Lie groupoid equipped with additional metric structure, which takes values in the K-theory of the Roe C*-algebra. We apply our theory to derive a Lichnerowicz type vanishing result for foliations on open manifolds.

JOSEPH VARILLY, Universidad de Costa Rica
How does chirality of the Standard Model arise?

Noncommutative geometry has proposed principles leading to interaction patterns close to the observed structure of the Standard Model of elementary particles, with its usual gauge group, but chirality is taken as an input. It turns out that an alternative framework, that of string-local quantum fields, enables one to deduce chirality of the electroweak sector from tree-level renormalizability at second order, on replacing gauge invariance by string independence.

RUFUS WILLETT, University of Hawaii
Cartans and rigidity for uniform Roe algebras

Uniform Roe algebras are C*-algebras associated to (discrete) metric spaces. They can also be thought of as the C*-algebras of certain étale groupoids, and as such have a natural Cartan subalgebra. I'll discuss uniqueness of this Cartan, and some corresponding rigidity results for the associated uniform Roe algebras: these imply in particular that the space can in some sense be recovered from the algebra. This is based on joint work with Jan Spakula, and with Stuart White.

MITSURU WILSON, Universidad de los Andes, Bogota, Colombia
Canonical group quantization of the noncommutative tori and the noncommutative spheres

We construct a noncommutative generalization of canonical group quantization. First, I will explain how this framework of quantization is constructed by considering a Lie group G acting by symplectic transformation on a symplectic manifold M and by finding a map $P : \mathfrak{g} \to C^\infty(M)$. I will then explain how 2-cocycles enters into the quantization picture as obstructions. All of these will be demonstrated using the noncommutative tori and the noncommutative spheres as examples.

FERESHTEH YAZDANI, University of NewBrunswick
Hopf-Cyclic Cohomology of $\mathcal{H}_n$ with Nontrivial Coefficients

We show that the space Ω_n^q of formal differential q-forms on $\mathbb{R}^n$ has an (induced) SAYD module structure on the Connes-Moscovici Hopf algebra $\mathcal{H}_n$. We thus identify the Hopf-cyclic cohomology $\mathcal{H}_n$ with coefficients in formal differential forms with the Gelfand-Fuks cohomology of the Lie algebra W_n of formal vector fields on $\mathbb{R}^n$. Furthermore, we introduce a multiplicative structure on the Hopf-cyclic bicomplex, and we show that this van Est-type isomorphism is multiplicative. We finally illustrate the whole machinery in the case $n = 1$ and we see that the Hopf cocycles are continuous and not algebraic ones.