Nonlinear Dispersive Equations
Équations dispersives non linéaires

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WALTER CRAIG, McMaster University

Birkhoff normal form for nonlinear wave equations

Many theorems on global existence of small amplitude solutions of nonlinear wave equations in $\mathbb{R}^n$ depend upon a competition between the time decay of solutions and the degree of the nonlinearity. Decay estimates are more effective when inessential nonlinear terms are able to be removed through a well-chosen transformation. Additionally, most physically relevant wave equations can be formulated as Hamiltonian PDEs, and the analysis of their solutions can be considered in this context. In this talk, we construct Birkhoff normal forms transformations for the class of wave equations which are Hamiltonian PDEs and null forms, using the flow of an auxiliary Hamiltonian system . This gives a new proof via canonical transformations of the global existence theorems for null form wave equations of S. Klainerman and J. Shatah in space dimensions $n \geq 3$. The case $n = 2$ is also under consideration, which additionally involves a normal forms interpretation of modified scattering. These results are work-in-progress with A. French and C.-R. Yang.

MAGDALENA CZUBAK, University of Colorado at Boulder

The exterior domain problem on the hyperbolic plane

In this talk we discuss the steady Navier-Stokes problem for the exterior domain in the hyperbolic plane. The problem has a satisfactory answer in three dimensions in the Euclidean setting, but there are questions that remain open in two dimensions, and they have been open since the work of Leray. We show these questions can be answered if we pose them on the hyperbolic plane.

LUIZ GUSTAVO FARAH, Universidade Federal de Minas Gerais (UFMG) - Brazil

Instability of solitary waves in the KdV-type equations - Part I

We revisit the classical result of Martel-Merle about instability of solitary waves for the critical KdV equation. Then we discuss how those techniques can be used to study the similar phenomenon of instability in higher dimensional generalizations of the KdV equation. This is a joint work with Justin Holmer and Svetlana Roudenko.

ALEX HIMONAS, University of Notre Dame

Ill-posedness for a family of nonlinear and nonlocal evolution equations

We shall discuss the well-posedness of a family of nonlinear and nonlocal evolution equations that includes the Camassa-Holm (CH), the Degasperis-Procesi (DP), and the Novikov equations. In Sobolev spaces H^s with $s < 3/2$, we construct 2-peakon solutions that collide in finite time in a such a way that both the initial profile and the collision time are arbitrarily small. However, at the collision time the H^s norm of the solution is arbitrarily large when $s < 3/2$ but close to 3/2 thus resulting to norm inflation and ill-posedness. For the remaining Sobolev exponents we prove non-uniqueness. Considering that these equations are well-posed for $s > 3/2$, these results establish $3/2$ as the critical index of their well-posedness. This is work in collaboration with Curtis Holliman and Carlos Kenig.

MIHAELA IFRIM, UC, Berkeley, USA

Well-posedness and dispersive decay of small data solutions for the Benjamin-Ono equation

Our goal is to take a first step toward understanding the long time dynamics of solutions for the Benjamin-Ono equation. While this problem is known to be both completely integrable and globally well-posed in L2, much less seems to be known concerning its long time dynamics. We present that for small localized data the solutions have (nearly) dispersive dynamics almost globally in time. An additional objective is to revisit the L2 theory for the Benjamin-Ono equation and provide a simpler, self-contained approach. This is joined work with Daniel Tataru.

DANA MENDELSON, University of Chicago

An infinite sequence of conserved quantities for the cubic Gross-Pitaevskii hierarchy on R

We consider the (de)focusing cubic Gross-Pitaevskii (GP) hierarchy on R, which is an infinite hierarchy of coupled linear non-homogeneous PDE which appears in the derivation of the cubic nonlinear Schrodinger (NLS) equation from quantum many-particle systems. Motivated by the fact that the cubic NLS on R is an integrable equation which admits infinitely many conserved quantities, we exhibit an infinite sequence of operators which generate analogous conserved quantities for the GP hierarchy. This is joint work with Andrea Nahmod, Natasa Pavlovic, and Gigliola Staffilani.

CLAUDIO MUÑOZ, University of Chile

Decay of small perturbations on 1D scalar field equations

The purpose of this talk is to show that small perturbations of some one dimensional nonlinear scalar field equations must decay according to a particular regime depending on the parity of the initial data. We prove these results by using well-chosen Virial identities, a technique adapted to the natural energy space of the problem. This is joint work with Michal Kowalczyk and Yvan Martel.

ANDREA NAHMOD, University of Massachusetts Amherst

Probabilistic well-posedness for 2D wave equations with derivative null form nonlinearity.

We will first explain some of the ideas behind randomization and dynamics in nonlinear wave and dispersive PDE. We then describe recent work of myself joint with Chanillo, Czubak, Mendelson and Staffilani in which we treat probabilistic well-posedness of a geometric wave equation with randomized supercritical data.

DIDIER PILOD, Universidade Federal do Rio de Janeiro

Construction of a minimal mass blow up solution of the modified Benjamin-Ono equation

We construct a minimal mass blow up solution of the modified Benjamin-Ono equation (mBO), which is a classical one dimensional nonlinear dispersive model.

Let $Q \in H^{\frac{1}{2}}$, $Q > 0$, be the unique ground state solution associated to mBO. We show the existence of a solution S of mBO satisfying $\|S\|_{L^2} = \|Q\|_{L^2}$ and

$$S(t) - \frac{1}{\lambda^{\frac{1}{2}}(t)} Q\left(\frac{\cdot - x(t)}{\lambda(t)}\right) \to 0 \quad \text{in } \ H^{\frac{1}{2}}(\mathbb{R}) \text{ as } \ t \downarrow 0,$$

where

$$\lambda(t) \sim t, \quad x(t) \sim -|\ln t| \quad \text{and} \quad \|S(t)\|_{\dot{H}^{\frac{1}{2}}} \sim t^{-\frac{1}{2}} \|Q\|_{\dot{H}^{\frac{1}{2}}} \quad \text{as } t \downarrow 0.$$

This existence result is analogous to the one obtained by Martel, Merle and Raphael (J. Eur. Math. Soc., 17 (2015)) for the mass critical generalized Korteweg-de Vries equation (gKdV). However, in contrast with the gKdV equation, for which the blow up problem is now well-understood in a neighborhood of the ground state, S is the first example of blow up solution for mBO.

The proof involves the construction of a blow up profile, energy estimates as well as refined localization arguments, developed in the context of Benjamin-Ono type equations by Kenig, Martel and Robbiano (Ann. Inst. H. Poincaré, Anal. Non Lin., 28 (2011)). Due to the lack of information on the mBO flow around the ground state, the energy estimates have to be considerably sharpened here.

This talk is based on a joint work with Yvan Martel (Ecole Polytechnique)

SVETLANA ROUDENKO, The George Washington University
Instability of solitary waves in the KdV-type equations - Part II

We revisit the classical result of Martel-Merle about instability of solitary waves for the critical KdV equation. Then we discuss how those techniques can be used to study the similar phenomenon of instability in higher dimensional generalizations of the KdV equation. This is a joint work with Luiz Farah and Justin Holmer.

LUIS VEGA, BCAM, Spain
The Talbot effect and the dynamics of vortex filaments: transfer of energy and momentum

I shall present some recent work done in collaboration with V. Banica and F. De La Hoz about the evolution of vortex filaments according to so called binormal flow. I will exhibit a non-linear Talbot effect and some results concerning the transfer of energy and of linear momentum.