Mathematics of Quantum Phases of Matter and Quantum Information
Mathématiques des phases quantiques de la matière et de l'information quantique
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ALONSO BOTERO, Universidad de los Andes
Asymptotic entanglement concentration of multi-qubit W states

While the general problem of multipartite entanglement concentration remains open, one can still show that for a certain class of multipartite states, the multi-qubit W class, it is possible to asymptotically concentrate the entanglement of n copies of the state into a single canonical "completely entangled" state (i.e., with maximally mixed one-particle density matrices), between local Hilbert spaces of high dimension. In this talk we show how this remarkable property of W states follows from the application of a multi-local Schur transform, permutation symmetry, and the algebra of SLOCC covaraints for W states, and how it in turn admits a concentration protocol akin to the distortion-free protocol of Hayashi and Matsumoto. If time permits, we will also discuss how the asymptotic tail exponents can be computed in principle, and discuss some partial results in this respect.

ALEX BULLIVANT, University of Leeds
Higher Symmetry Topological Phases and Loop Braid Invariants

The non-Abelian exchange statistics of anyons in 2+1D topological phases of matter have received considerable interest in recent years due to the potential applications in topological quantum computation. Utilising the connection between topological phases of matter and extended Topological Quantum Field Theories (TQFT) has provided a powerful framework for understanding physically realistic models for realising anyons in condensed matter systems and providing representations of the braid group, describing the exchange of point particles.

In 3+1D it is well known that the exchange of point particles is necessarily described by Bosonic or Fermionic exchange statistics. However, it has been shown that many models of 3+1D topological phases of matter naturally support loop-like excitations. When one loop is passed through another, these excitations support possibly non-Abelian exchange statistics generating the so called loop-braid group.

In this talk I will outline a class of candidate 3+1D topological phases of matter arising from considerations in Higher Lattice Gauge Theory (HLGT)[1606.06639,1702.00868]. Such theories generalise Lattice Gauge Theories (LGT) with finite gauge group to the case of finite gauge 2-group which are described by crossed modules. I will then utilise this construction and ideas from extended TQFT to define representations of loop-braids within the HLGT framework.

FIONA BURNELL, University of Minnesota
Hamiltonians for anyon permuting symmetries

Commuting projector Hamiltonians are closely related to discrete topological invariants. I will describe a general construction for commuting projector Hamiltonians that realize a global anyon-permuting symmetry in an on-site manner, and discuss their relationship with the gauged (equivariantized) version of these topological orders (and commuting projector Hamiltonians).

XIE CHEN, California Institute of Technology
Group Cohomology and Symmetry Protected Topological Phases

The mathematical concept of Group Cohomology has recently played an important role in condensed matter physics, in the study of Symmetry Protected Topological Phases. In this talk, I am going to discuss the development in this field. In particular, I am going to discuss how group cocycles can be used to construct condensed matter models with global symmetry and how nontrivial group cocycle leads to models with nontrivial physical effect when the system is placed on a manifold with boundary. Open questions and future directions are also discussed.

XINGSHAN CUI, Stanford University

State Sum Invariants of Three Manifolds from Spherical Multi-fusion Categories

We define a family of quantum invariants of closed oriented 3-manifolds using spherical multi-fusion categories. The state sum nature of this invariant leads directly to $(2+1)$-dimensional topological quantum field theories (TQFTs), which generalize the Turaev-Viro-Barrett-Westbury (TVBW) TQFTs from spherical fusion categories. The invariant is given as a state sum over labeled triangulations, which is mostly parallel to, but richer than the TVBW approach in that here the labels live not only on 1-simplices but also on 0-simplices. It is shown that a multi-fusion category in general cannot be a spherical fusion category in the usual sense. Thus we introduce the concept of a spherical multi-fusion category by imposing a weakened version of sphericity. Besides containing the TVBW theory, our construction also includes the recent higher gauge theory $(2+1)$-TQFTs given by Kapustin and Thorngren, which was not known to have a categorical origin before.

FRANCISCO DELGADO, Tecnologico de Monterrey

SU(2) decomposition for complex quantum information hamiltonians generating entanglement

In the gate array version of Quantum Computation, the use of convenient and appropriate gates is essential. But while involved gates adopt convenient forms for the computational algorithms, in the practice, their design depends on specific quantum systems and physical interactions involved. Quantum resources and gates design are restricted to properties and limitations imposed by the physical elements considered in the set up. In addition, predictable and controllable manipulation procedures should be addressed on them. On this scenario, two level quantum systems are the basic elements to connect the binary nature of classical computation with quantum computation. This work presents a general approach to set control procedures by decomposing the dynamics in $SU(2^{2d})$ for $2d$-partite two level spin systems including entangling operations into 2^{2d-1} $SU(2)$ subsystems for the generalized Hamiltonian:

$$\tilde{H} = \sum_{\{i_k\}} h_{\{i_k\}} \bigotimes_{k=1}^{n} \sigma_{i_k} = \sum_{\mathcal{I}=0}^{4^n-1} h_{\mathcal{I}_4^n} \bigotimes_{k=1}^{n} \sigma_{\mathcal{I}_{4,k}^n}$$

by expressing the dynamics on proper basis: the generalized Bell states. Thus, binary operations naturally arise there. Still, alternating the directions of local (or possibly non-local) interaction terms in the Hamiltonian, the procedure states a universal exchange semantics on that basis. Thus, the structure developed can be understood as the splitting of the $2d$ physical systems in 2^{2d-1} pairs of 2 level information channels.

DMITRI NIKSHYCH, University of New Hampshire

Classifying braidings on fusion categories

It is well known that braidings on a fusion category C are in bijection with sections of the forgetful functor $Z(C) \to C$, where $Z(C)$ is the center of C. We extend this observation by proving that braidings on fusion categories Morita equivalent to C are parameterized by pairs (A, S) consisting of a Lagrangian algebra A in $Z(C)$ and a fusion subcategory $S \subset Z(C)$ transversal to A and such that $\dim(S) = \dim(C)$. We use this to classify braidings on group-theoretical and on factorizable fusion categories. We discuss a relation between these results and the Belavin-Drinfeld classification of r-matrices for simple Lie algebras.

MARY BETH RUSKAI, University of Vermont

Extreme Points of Unital Quantum Channels

Several new classes of extreme points of unital and trace-preserving completely positive (CP) maps are analyzed. One class is not extreme in either the convex set of unital CP maps or the set of trace-preserving CP maps and is factorizable. Another class is extreme for both the set of unital CP maps and the set of trace-preserving CP maps, except for certain critical parameters. For those parameters the linear dependence of the matrices in the Choi product condition are associated with representations of the symmetric group.

Joint work with U. Haagerup and M. Musat

PAULO TEOTONIO-SOBRINHO, Instituto de Fisica - São Paulo University
Topological Order in Higher Gauge Theories and Cohomology

Quantum double models are good examples of quantum systems with topological order. They are lattice gauge theories with a finite gauge group. The lattice is described by a finite simplicial complex X whereas gauge configurations are maps from the set of 1-simplices into the gauge group. Recently, higher gauge theory generalizations have appeared in the literature. Examples where a 2-group plays the role of the usual gauge group lead to interesting models and new topological phases.

In this talk we are going to present a model with topological order that generalizes some of the ideas above. In our construction, the lattice is replaced by a chain complex C of finitely generated free abelian groups and the gauge group is replaced by a chain complex G of finite abelian groups. From this initial data, we construct a Hilbert space $\mathcal{H}$ and a frustration free Hamiltonian. Topological order is manifest when we describe the ground state space $\mathcal{H}_0 \subset \mathcal{H}$ in terms of a cohomology with coefficients in a finite chain complex. This cohomology, denoted by $H^0(C, G)$, was first introduced by Ronald Brown in 1964. He proved that H^0 is isomorphic to a product of usual cohomology groups. This results allows us to compute the ground state degeneracy and to find a convenient set of quantum numbers that labels the states of $\mathcal{H}_0$. Abelian examples of 1-gauge and 2-gauge theories are recovered as special cases.

SALVADOR VENEGAS-ANDRACA, Tecnologico de Monterrey, Escuela de Ingenieria y Ciencias, Mexico
Entanglement-tuned quantum walks

Quantum walks were originally developed as quantum-mechanical counterparts of classical random walks. In the early days of this cross-disciplinary research field, quantum walks were used just as a mathematical tool to develop sophisticated algorithms. Later on and in stark contrast to the algorithmic properties of classical random walks, it was proved that quantum walks constitute a universal model for quantum computation.

Quantum entanglement is expected to play a key role in the formulation of quantum algorithms that are faster than their classical counterparts. Although some aspects of quantum entanglement and computational speed up have been studied, we lack a general framework to explicitly manipulate quantum entanglement for building quantum algorithms, being the ultimate purpose of such framework to provide quantum programmers with mathematical descriptions and subroutines in order to build algorithms for solving complex problems.

The role of entanglement in quantum walks and quantum walk-based algorithms is an open area of research. Motivated by the wish to further study the effect of entanglement in discrete quantum walks, in this paper we shall present a preliminary framework for manipulating quantum walk parameters via quantum entanglement together with a succinct yet complete review of the state-of-the-art on the role of quantum entanglement in the definition and dynamics of quantum walks.

DOMINIC WILLIAMSON, University of Vienna
Hamiltonian models for topological phases of matter in three spatial dimensions

We present commuting projector Hamiltonian realizations of a large class of (3+1)D topological models based on unitary G-crossed braided fusion categories. This construction comes with a wealth of examples from the physics literature on symmetry-enriched topological phases. The spacetime counterparts to our Hamiltonians are a family of unitary state sum topological quantum fields theories (TQFTs), recently defined by Cui, that appear to capture all known constructions in the literature, including the Crane-Yetter-Walker-Wang and 2-Group gauge theory models. We also present Hamiltonian realizations of another state sum TQFT family, recently constructed by Kashaev, whose relation to existing models was previously unknown.

We argue that these TQFTs are captured as a special case of the Crane-Yetter-Walker-Wang model, with a premodular input category in some instances.

JON YARD, University of Waterloo and Perimeter Institute

Topological phases and arithmetic

In this talk, I will present various applications of arithmetic to the study of topological phases. I will first discuss joint work illustrating the role played by different kinds of equivalence classes of quadratic forms in the classification of abelian topological phases. I will also present examples of arithmetic groups arising from images of braid group representations associated to certain nonabelian phases.

QING ZHANG, Texas A&M University

Congruence Subgroups and Super-Modular Categories

A super-modular category is a unitary ribbon fusion category with Müger center equivalent to the unitary symmetric ribbon category of super-vector spaces. For modular categories, Ng and Schauenburg showed that the kernel of the associated projective representation of the modular group is a congruence subgroup. In the super-modular setting, one gets a representation of the theta subgroup of the modular group by taking the fermionic modular quotient. It is conjectured that the kernel of this representation is also congruence of some level. We verify this for any super-modular category having a minimal modular extension. We also provide evidence for the conjecture by looking at super-modular categories arising from quantum groups at roots of unity.