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**MCA Prize Lectures**  
**Conférences prix du CMA**

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**UMBERTO HRYNIEWICZ**, Universidade Federal do Rio de Janeiro, Brazil

*Symplectic dynamics: methods and results*

Poincaré's last geometric theorem led to the Arnold conjectures, which are deeply related to the birth of symplectic topology. The first goal of this talk is to review parts of this beautiful piece of history of mathematics, leading to the introduction of the term 'Symplectic Dynamics' by Hofer. Then I will move on to explain how modern methods in symplectic topology, namely holomorphic curves and Eliashberg-Givental-Hofer's Symplectic Field Theory, can be used to study Hamiltonian dynamics in the large. I will focus on the existence of global surfaces of section, and on versions of the Poincaré-Birkhoff theorem for Reeb flows.

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**ROBERT MORRIS**, Instituto de Matemática Pura e Aplicada, Brazil

*Random graph processes*

For the past several decades, since Erdős' 1947 lower bound on the Ramsey numbers  $R(k)$ , randomness has been an important and powerful tool for demonstrating the existence of counter-intuitive objects. When dealing with random objects, it is often useful to reveal the randomness gradually, rather than all at once; that is, to turn a static random object into a random process. In the 1980s and 1990s, several important techniques for studying the evolution of such processes were introduced by Bollobás, Rödl, Rucinski, Wormald, and others, and in recent years these techniques have been developed further by a number of different authors, and have been used to resolve several well-known open problems. In this talk we will describe a few of these recent developments, focusing our attention on two or three specific examples. In particular, we will discuss the Ramsey numbers  $R(3, k)$ , and a problem of Pomerance about the existence in a random set of integers of a subset whose product is a square. In each case, the key to the proof is controlling the evolution of a suitably-chosen random (hyper)graph process using self-correcting martingales. This talk is based on joint work with Paul Balister, Béla Bollobás, Gonzalo Fiz Pontiveros, Simon Griffiths and Paul Smith.

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**PABLO SHMERKIN**, Universidad de Buenos Aires, Argentina

*Expansions in bases 2 and 3: old conjectures and new results*

In the 1960s, H. Furstenberg proposed a series of conjectures that, in different ways, aim to capture the heuristic principle that "expansions in bases 2 and 3 have no common structure". Some of these conjectures remain wide open, but several have been solved in the last few years. In the talk I will survey several of the conjectures, and then focus on Furstenberg's conjecture on intersections of Cantor sets, which has been recently settled independently by Meng Wu and by myself (with two strikingly different proofs). I will aim to make the talk accessible to a general audience of mathematicians; some basic concepts in ergodic theory and fractal geometry will be introduced along the way.

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**HÉCTOR H. PASTÉN VÁSQUEZ**, Harvard University, USA

*The abc Conjecture and the  $d(abc)$  Theorem*

The abc Conjecture is a central open problem in number theory. It has a wide range of surprising applications, although unconditional progress in the direction of a solution has been difficult and modest. In this talk I will recall some consequences of the abc Conjecture and I will review the existing approaches and unconditional partial progress towards it. Then, I will discuss recent methods that lead to the proof of a new unconditional result in this setting, the  $d(abc)$  Theorem.

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**VLAD VICOL**, Princeton University, USA

*Turbulent weak solutions of the Euler equations*

Motivated by Kolmogorov's theory of hydrodynamic turbulence, we consider dissipative weak solutions to the 3D incompressible Euler equations. We show that there exist infinitely many weak solutions of the 3D Euler equations, which are continuous in time, lie in the Sobolev space  $H^s$  with respect to space, and they do not conserve the kinetic energy. Here the smoothness parameter  $s$  is any number less than  $5/14$ . In particular, this exponent lies above the Onsager critical value  $1/3$  consistent with Kolmogorov's  $-4/5$  law for the third order structure functions. We shall also discuss bounds for the second order structure functions, which deviate from the classical Kolmogorov 1941 theory.