Mathematical Physics
Physique mathématique

(Org: **Rafael Benguria** (Universidad Catolica de Chile), **Vojkan Jaksic** (McGill University) and/et **Robert Seiringer** (IST))

GUISEPPE DE NITTIS, Pontificia Universidad Católica de Chile
Linear Response Theory: An Analytic-Algebraic Approach

Linear response theory (LRT) is a tool with which one can study the response of systems that are driven out of equilibrium by external perturbations. In this talk I present a modern and systematic approach to LRT by combining analytic and algebraic ideas. The theory is robust and provides a tool to implement LRT for a wide array of systems like periodic and random systems in the discrete and the continuum. The mathematical framework of the theory is outlined firstly: the relevant von Neumann algebras, noncommutative L^p- and Sobolev spaces are introduced; the notion of isopectral perturbations and the associated dynamics and commutators are studied; their construction is then made explicit for various physical systems (quantum systems, classical waves). The final part is dedicated to a presentation of the proofs of the Kubo and Kubo-Streda formulas.

ALESSANDRO GIULIANI, Università degli Studi Roma Tre
Haldane relation for interacting dimers

In this talk, we present some recent results on the existence and nature of the scaling limit of interacting, close-packed, dimers on the two-dimensional square lattice. By constructive Renormalization Group techniques, we compute: the multipoint dimer correlations, which display non-trivial critical exponents, continuously varying with the interaction strength; and the height fluctuations, which, after proper coarse graining and rescaling, converge to the massless Gaussian field with a suitable interaction-dependent pre-factor ('amplitude'). We also prove the identity between the critical exponent of the two-point dimer correlation and the amplitude of this massless Gaussian field. This identity is the restatement, in the context of interacting dimers, of one of the Haldane universality relations, part of his Luttinger liquid conjecture, originally formulated in the context of one-dimensional interacting Fermi systems. Joint work with V. Mastropietro and F. Toninelli.

GIAN MICHELE GRAF, ETH Zurich
An overview on topological insulators

Topological insulators are materials, which are conducting at their edges, though not in the bulk. Their essential physical properties take the form of an index, often associated to the Hamiltonian. "Topological" simply refers to the fact that indices remain invariant under continuous changes. Earliest examples occurred in connection with the Quantum Hall effect, which has been the source of various mathematical developments. In the last decade the concept has been refined by conditioning it to symmetries, such as time-reversal invariance. It has also been extended in part beyond band insulators, so as to include the localization regime as well. A general property of the indices, which should be preserved by any generalization, is bulk-edge correspondence, by which they should admit dual formulations, that is in terms of either the bulk or the edge properties of the material. The discussion is based on examples in dimension 1 and 2 exhibiting different symmetries.

ELLIOTT LIEB, Princeton University
Strichartz Inequality for Orthonormal Functions

We prove a Strichartz inequality for a system of orthonormal functions, with an optimal behavior of the constant in the limit of a large number of functions. The estimate generalizes the usual Strichartz inequality, in the same fashion as the Lieb-Thirring inequality generalizes the Sobolev inequality. Similarly, it generalizes the norms of Riesz and Bessel potentials, which will be recalled in the talk. As an application, we consider the Schroedinger equation in a time-dependent potential and we show the existence of the wave operator in the strong sense of a finite Schatten space norm. (joint work with R. Frank, M. Lewin and R. Seiringer)

MILIVOJE LUKIC, Rice University
Analytic quasiperiodic Schrodinger operators at small coupling

We study Schrödinger operators $H = -\frac{d^2}{dx^2} + V$ with quasiperiodic potentials V in the regime of analytic sampling function and small coupling. We prove that elements of its isospectral torus are also quasi-periodic Schrödinger operators in the same regime. We also prove almost periodicity of solutions of the KdV equation $\partial_t u - 6u\partial_x u + \partial_x^3 u = 0$ with this class of initial data $u(x,0) = V(x)$. The talk describes joint work with Ilia Binder, David Damanik, and Michael Goldstein.

JEFF SCHENKER, Michigan State University
Localization in disordered polaron models

Polaron models describe the motion of a tracer particle interacting with a quantum field. Various tight-binding polaron models with on-site disorder in tracer the particle potential will be discussed. These models provide a framework in which it is possible to realize some ideas related to many-body localization, in particular Fock-space localization bounds. Provided the hopping amplitude for the particle is small, we are able to prove localization for matrix elements of the resolvent, in particle position and in the field Fock space. These bounds imply a form of dynamical localization for the particle position that leaves open the possibility of resonant tunneling in Fock space between equivalent field configurations. Some open problems regarding the character of high energy eigenstates will be discussed. (Joint work with Rajinder Mavi.)

BENJAMIN SCHLEIN, University of Zurich
Dynamical and spectral properties of Bose-Einstein condensates

We consider systems of N bosons in the Gross-Pitaevskii regime. We present new techniques that allow us to prove the convergence towards the time-dependent Gross-Pitaevskii equation with optimal rate. Furthermore, we explain how, for small potentials, this approach can be used to show complete Bose-Einstein condensation (with a uniform bound on the number of excitations) for low energy states. For scaling limits interpolating between the mean-field and the Gross-Pitaevskii regime, the same method can also be used to establish the validity of Bogoliubov theory for the low-lying excitation spectrum. This talk is based on joint works with C. Boccato, C. Brennecke and S. Cenatiempo.

ISRAEL MICHAEL SIGAL, University of Toronto
On the Bogolubov-de Gennes Equations of Superconductivity

In this talk, I describe recent results on the Bogolubov-de Gennes equations. These equations give an equivalent formulation of the BCS theory of superconductivity. I discuss general features of the equations and key physical classes of solutions (normal, superconducting, vortex and vortex lattice states). I describe results on existence of the normal, superconducting and vortex lattice states for non-zero magnetic fields and stability/instability of the normal states for large/small temperature or/and magnetic fields.

JAN PHILIP SOLOVAJ, University of Copenhagen
Dirac operators with magnetic links

The occurence of zero modes for Dirac operators with magnetic fields is the cause of break down of stability of matter for charged systems. All known examples of magnetic fields leading to zero modes are geometrically very complex. In order to better understand this geometry I will discuss singular magnetic fields supported on a finite number of possibly interlinking field lines (magnetic links). I will show that the occurence of zero modes is intimately connected to the twisting and interlinking of the field lines. The result will rely on explicitly calculating appropriate spectral flows for the Dirac operators. This is joint work with Fabian Portmann and Jeremy Sok.

RAFAEL TIEDRA, Pontifical Catholic University of Chile
Spectral analysis of quantum walks with an anisotropic coin

We perform the spectral analysis of the evolution operator U of quantum walks with an anisotropic coin, which include one-defect models, two-phase quantum walks, and topological phase quantum walks as special cases. In particular, we determine the essential spectrum of U, we show the existence of locally U-smooth operators, we prove the discreteness of the eigenvalues of U outside the thresholds, and we prove the absence of singular continuous spectrum for U. Our analysis is based on new commutator methods for unitary operators in a two-Hilbert spaces setting, which are of independent interest.

This is a joint work with Serge Richard (Nagoya University) and Akito Suzuki (Shinshu University).

JULIO TOLOZA, Universidad Nacional del Sur (Argentina)
Dispersion Estimates for Spherical Schrödinger Operators

In this talk I present some results concerning a dispersion estimate for one-dimensional perturbed radial Schrödinger operators.

This is a joint work with A. Kostenko and G. Teschl (Univ. Vienna).

HANNE VAN DEN BOSCH, Pontificia Universidad Católica de Chile
Spectrum of Dirac operators describing Graphene Quantum dots

Low energy electronic excitations in graphene, a two-dimensional lattice of carbon atoms, are described effectively by a two–dimensional Dirac operator. For a bounded flake of graphene (a quantum dot), the choice of boundary conditions determines various properties of the spectrum. Several of these choices appear in the physics literature on graphene. For a simply connected flake and a family of boundary conditions, we obtain an explicit lower bound on the spectral gap around zero. We can also study the effect of the boundary conditions on eigenvalue sums in the semiclassical limit. This is joint work with Rafael Benguria, Søren Fournais and Edgardo Stockmeyer.