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## Motives and Periods

### Périodes et motifs

(Org: **Charles Doran** (University of Alberta & University of Maryland), **Matt Kerr** (Washington University) and/et **James Lewis** (University of Alberta))

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**PATRICK BROSANAN**, University of Maryland, College Park

*Perverse obstructions to regular flat compactifications*

Suppose  $S$  is a smooth, complex variety containing a dense Zariski open subset  $U$ , and suppose  $W$  is a smooth projective family of varieties over  $U$ . It seems natural to ask when  $W$  admits a regular flat compactification over  $S$ . In other words, when does there exist a smooth variety  $X$  flat and proper over  $S$  containing  $W$  as a Zariski open subset? Using resolution of singularities, it is not hard to see that it is always possible to find a regular flat compactification when  $S$  is a curve. My main goal is to point out that, when  $\dim S \geq 1$ , there are obstructions coming from local intersection cohomology. My main motivation is a recent paper of Laza, Sacca and Voisin (LSV) who construct a regular flat compactification in the case that  $W$  is a certain family of abelian 5-folds over an open subset of 5 dimensional projective space. On the one hand, I'll explain how to compute the intersection cohomology in certain related examples and show that these are obstructed. On the other hand, I'll use the vanishing of the intersection cohomology obstructions implied by the LSV theorem to deduce a theorem on the palindromicity (=numerical Poincare duality) of the cohomology groups of certain singular cubic 3-folds.

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**PEDRO LUIS DEL ANGEL**, Centro de Investigación en Matemáticas A.C.

*Specialization of cycles and the K-theory elevator*

A general specialization map is constructed for higher Chow groups and used to prove a going-up theorem for algebraic cycles in the setting of nodal degenerations. The constructions are applied in two different directions. On the one hand, it is shown how the modified diagonal of Gross and Schoen on a non-hyperelliptic curve of genus three, which degenerates to a nodal curve, gives rise to a higher indecomposable cycle on a genus two curve. On the other hand, we also explore a similar phenomenon in the setting of normal functions and the image of the Abel-Jacobi map.

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**CHARLES DORAN**, University of Alberta and ICERM

*Hodge Numbers from Picard-Fuchs Equations*

Given a variation of Hodge structure over  $\mathbb{P}^1$  with Hodge numbers  $(1, 1, \dots, 1)$ , we show how to compute the degrees of the Deligne extension of its Hodge bundles, following Eskin-Kontsevich-Möller-Zorich, by using the local exponents of the corresponding Picard-Fuchs equation. This allows us to compute the Hodge numbers of Zucker's Hodge structure on the corresponding parabolic cohomology groups. We also apply this to families of elliptic curves, K3 surfaces and Calabi-Yau threefolds. This is joint work with Andrew Harder and Alan Thompson.

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**PHILLIP GRIFFITHS**, Institute for Advanced Study

*Hodge Theory and Moduli*

The equivalence classes of smooth algebraic varieties  $X$  of a particular type form its moduli space  $\mathcal{M}$ , and their study is a central problem in algebraic geometry. When  $X$  is of general type  $\mathcal{M}$  exists and has a canonical compactification  $\overline{\mathcal{M}}$  as a projective algebraic variety. Aside from a few classical cases (curves, K3 surfaces, abelian varieties) very little is known about the boundary  $\partial\mathcal{M} = \overline{\mathcal{M}} \setminus \mathcal{M}$  and the singular varieties  $X_0$  that corresponds to boundary points. In this talk we will explain how Hodge theory provides basic invariants of the  $X_0$ 's and in some early examples may be used to help understand geometrically the boundary structure of moduli.

Specifically, we will discuss how to define the Satake-Baily-Borel compactification  $\overline{\mathcal{M}}_{SBB} = Proj \Omega_e$  of the image of the period mapping of  $\mathcal{M}$  where  $\Omega \rightarrow \overline{\mathcal{M}}$  is the canonically extended Hodge bundle. We will also discuss the examples where  $X$

is a regular surface with  $p_g(X) = 2$  and  $K_X^2 = 1, 2$ ; in these examples the boundary strata of  $\partial M = \overline{\mathcal{M}}_{SBB} \setminus \mathcal{M}$  classify the Gorenstein degenerations of the  $X_0$ 's. The above is joint work with Green, Laza and Robles, and the  $K_X^2 = 1$  case uses recent results of Franciosi-Pardini and Rollenske.

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**ANDREW HARDER**, University of Miami  
*Hodge numbers of Landau-Ginzburg models*

Mirror symmetry predicts that  $d$ -dimensional Calabi-Yau manifolds should come in pairs  $X$  and  $X^\vee$  which, among other things, satisfy

$$h^{p,q}(X) = h^{d-p,q}(X^\vee).$$

Mirror symmetry also predicts that Fano manifolds admit mirror partners which are pairs  $(Y, w)$  where  $Y$  is a quasiprojective variety and  $w$  is a regular function on  $Y$ . Recently, Katzarkov, Kontsevich and Pantev have conjectured that a subtle form of Hodge number duality holds between Fano manifolds and their mirrors which relates the Hodge numbers of Fano varieties to the cohomology of complexes of " $f$ -adapted logarithmic forms". I will discuss recent work which shows that the Hodge numbers of  $(Y, w)$  can be computed in terms of classical Hodge theory and I will show that in dimensions 2 and 3, these Hodge numbers have very concrete interpretations.

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**ANDREAS MALMENDIER**, Utah State University  
*Geometry of (1,2)-polarized Kummer surfaces*

A smooth K3 surface obtained as the blow-up of the quotient of a four-torus by the involution automorphism at all 16 fixed points is called a Kummer surface. Kummer surface need not be algebraic, just as the original torus need not be. However, algebraic Kummer surfaces obtained from abelian varieties provide a fascinating arena for string compactification as they are not trivial spaces but are sufficiently simple for one to be able to analyze most of their properties in detail.

In this talk, we give an explicit description for the relation between algebraic Kummer surfaces of Jacobians of genus-two curves with principal polarization and those associated to  $(1, 2)$ -polarized abelian surfaces from three different angles: the point of view of 1) the binational geometry of quartic surfaces in  $\mathbb{P}^3$  using even-eights, 2) elliptic fibrations on K3 surfaces of Picard-rank 17 over  $\mathbb{P}^1$  using Nikulin involutions, 3) theta-functions of genus-two using two-isogeny. Finally, we will explain how these  $(1,2)$ -polarized Kummer surfaces naturally allow for an identification of the complex gauge coupling in Seiberg-Witten gauge theory with the axion-dilaton modulus in string theory using an old idea of Sen. (This is joint work with Adrian Clingher.)

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**COLLEEN ROBLES**, Duke U  
*Generalizing the Satake-Bailey-Borel compactification*

I will report on work in progress to generalize the Satake-Bailey-Borel compactification of locally Hermitian symmetric domains to arbitrary arithmetic quotients of Mumford-Tate domains.