Models and Methods in Evolutionary Differential Equations on Mixed Scales
Modèles et méthodes en équations différentielles évolutives sur échelles mixtes
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HARBIR ANTIL, George Mason University
Fractional Operators with Inhomogeneous Boundary Conditions: Analysis, Control, and Discretization

In this talk we introduce new characterizations of spectral fractional Laplacian to incorporate nonhomogeneous Dirichlet and Neumann boundary conditions. The classical cases with homogeneous boundary conditions arise as a special case. We apply our definition to fractional elliptic equations of order $s \in (0,1)$ with nonzero Dirichlet and Neumann boundary conditions. Here the domain Ω is assumed to be a bounded, quasi-convex Lipschitz domain.

To impose the nonzero boundary conditions, we construct fractional harmonic extensions of the boundary data. It is shown that solving for the fractional harmonic extension is equivalent to solving for the standard harmonic extension in the very-weak form. The latter result is of independent interest as well. The remaining fractional elliptic problem (with homogeneous boundary data) can be realized using the existing techniques. We introduce finite element discretizations and derive discretization error estimates in natural norms, which are confirmed by numerical experiments. We also apply our characterizations to Dirichlet and Neumann boundary optimal control problems with fractional elliptic equation as constraints.

EVERALDO DE MELLO BONOTTO, University of São Paulo, Brazil

TOMÁS CARABALLO, Universidad de Sevilla
Asymptotic behavior of 2D-Navier-Stokes equations with bounded or unbounded delay

In this talk we exhibit different methods to analyze the asymptotic behavior of solutions to a 2D-Navier-Stokes model when the external force contains hereditary characteristics (constant, distributed or variable delay, memory, etc). First we provide some results on the existence and uniqueness of solutions. Next, the existence of stationary solution is established by Lax-Milgram theorem and Schauder fixed point theorem. Then the local stability analysis of stationary solution is studied by using the theory of Lyapunov functions, the Razumikhin-Lyapunov technique. In the end, Lyapunov functionals is also exploited some stability results. We highlight the differences in the asymptotic behavior in the particular case of bounded or unbounded variable delay.

MÁRCIA FEDERSON, Universidade de Sao Paulo
Generalized ODEs: overview and trends

We present an update of the latest results as well as applications to functional differential equations, measure differential equations and the Black-Scholes equation.

XIAOYING HAN, Auburn University, USA
Dynamical Structures in Stochastic Chemical Reaction Systems

Motivated by the need for dynamical analysis and model reduction in stiff stochastic chemical systems, we focus on the development of methodologies for analysis of the dynamical structure of singularly-perturbed stochastic dynamical systems. We outline a formulation based on random dynamical systems theory. We demonstrate the analysis for a model two-dimensional stochastic dynamical system built on an underlying deterministic system with a tailored fast-slow structure, and an analytically known slow manifold, employing multiplicative brownian motion noise forcing.

JIAYIN JIN, Georgia Tech
Dynamics near solitons of the supercritical gKDV equations

Consider generalized KDV equations with a power non-linearity $(u^p)_x$. These gKDV equations have solitary traveling waves, which are linearly unstable when p¿5 (supercritical case). Jointly with Zhiwu Lin and Chongchun Zeng, we constructed invariant manifolds (stable, unstable and center) near the orbits of the unstable traveling waves in the energy space. In particular, the local uniqueness and orbital stability of the center manifold is obtained. These invariant manifolds give a complete classification of the dynamics near unstable traveling waves.

JI LI, Huazhong University of Science and Technology, China

JAQUELINE G. MESQUITA, University of Brasilia
Massera's Theorem for dynamic equations on time scales and applications

In this talk, we prove Massera's type result for nonlinear dynamic equation on time scales defined on $\mathbb{R}$ and on infinite dimensional Banach space. Also, we prove that any almost periodic solution of dynamic equation on time scales is a p-periodic solution of this equation. These first results hold for any invariant under translations time scales. We also prove a version of Massera's theorem for linear and nonlinear q-difference equations. Finally, we provide some examples to illustrate our results. The main new results are parts of two joint works, one of them with Hernán Henríquez and Eduard Toon, and the other with Martin Bohner.

LAURENT MOONENS, Université Paris Sud 11, France

SON LUU NGUYEN, University of Puerto Rico, Rio Piedras
On the McKean-Vlasov limit for interacting diffusions with Markovian switching

This talk considers asymptotic properties of systems of weakly interacting particles with a common Markovian switching. It is shown that when the number of particles tends to infinity, certain measure-valued stochastic processes describing the time evolution of the systems converge to a stochastic nonlinear process which can be described as a solution to a stochastic McKean-Vlasov equation.

JULIANA PIMENTEL, Universidade Federal do ABC
Longtime behavior of reaction-diffusion equations with infinite-time blow-up

We account for the longtime behavior of solutions for a class of reaction-diffusion equations. In particular, we address those with global well-posedness but exhibiting blow-up in infinite time. The existence of unbounded trajectories requires the introduction of some objects interpreted as equilibria at infinity, yielding a more complex orbit structure than that appearing on dissipative systems. Under this setting, we still manage to extend known results and obtain a complete decomposition for the related unbounded global attractor.

RODRIGO PONCE, Universidad de Talca
Asymptotic behavior of solutions to a Volterra equation

In this talk we study the asymptotic behavior of solutions to the equation

$$\begin{cases} u'(t) = Au(t) + \displaystyle\int_0^t a(t-s)Au(s)ds, \ t \geq 0 \\ u(0) = x, \end{cases} \tag{1}$$

where $a(t) := \alpha \frac{t^{\mu-1}}{\Gamma(\mu)} e^{-\beta t}$, $\alpha, \beta, \mu \in \mathbb{R}$. Under appropriate assumptions on α, β, μ and A we prove that the solution u to equation (1) is uniform exponential stable, that is, there exist $C, \omega > 0$ such that for each $x \in D(A)$, the solution $\|u(t)\| \leq Ce^{-\omega t}\|x\|$ for all $t \geq 0$.

JOSEPH SHOMBERG, Providence College
On non-isothermal viscous nonlocal Cahn−Hilliard equations

We will discuss a non-isothermal viscous relaxation of some nonlocal Cahn−Hilliard equations. This perturbation problem generates a family of solution operators exhibiting dissipation and conservation. The solution operators admit a family of compact global attractors that are bounded in a more regular phase-space. An upper-semicontinuity result for this family of global attractors is also sought.

ANTONÍN SLAVÍK, Charles University (Prague, Czech Republic)
Invariant regions for systems of lattice reaction-diffusion equations

We study systems of lattice differential equations (i.e., equations with discrete space and continous time) of reaction-diffusion type. Such systems frequently appear in population dynamics (e.g., predator-prey models with diffusion).

After establishing some basic properties such as the local existence and global uniqueness of bounded solutions, we proceed to our main goal, which is the study of invariant regions. Our main result can be interpreted as an analogue of the weak maximum principle for systems of lattice differential equations. It is inspired by existing results for parabolic differential equations, but its proof is different and relies on the Euler approximations of solutions to lattice differential equations. As a corollary, we obtain a global existence theorem for nonlinear systems of lattice reaction-diffusion equations.

TAE-HYUK, Ted) Ahn (Saint Louis University, USA

LIJIN WANG, University of Chinese Academy of Sciences
Computational singular perturbation analysis of stochastic differential equations systems with stiffness

Computational singular perturbation (CSP) is a useful method for analysis, reduction, and time integration of stiff ordinary differential equation systems. It has found dominant utility, in particular, in chemical reaction systems with a large range of time scales at continuum and deterministic level. On the other hand, CSP is not directly applicable to chemical reaction systems at micro or meso-scale, where stochasticity plays a non- negligible role and thus has to be taken into account. In this work we develop a novel stochastic computational singular perturbation (SCSP) analysis and time integration framework, and associated algorithm, that can be used to not only construct efficiently the numerical solutions to stiff stochastic chemical reaction systems, but also analyze the dynamics of the reduced stochastic reaction systems. The algorithm is illustrated by an application to a benchmark stochastic differential equation model, and numerical experiments are carried out to demonstrate the effectiveness of the construction.