Morse, Conley, and Forman Approaches to Smooth and Discrete Dynamics
Approches de Morse, Conley et Forman en dynamique lisse et discrète
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BOGDAN BATKO, Jagiellonian University
Weak Index Pairs and the Conley Index for Discrete Multivalued Dynamical Systems

Motivation to revisit the Conley index theory for discrete multivalued dynamical systems [T. Kaczynski and M. Mrozek, Topology Appl., 65(1995), pp. 83–96] stems from the needs of broader real applications, in particular in the problem of reconstructing dynamics from samples or in combinatorial dynamics. We introduce a new, less restrictive definition of the isolating neighborhood. It turns out that then the main tool for the construction of the index, i.e., the index pair, is no longer useful. In order to overcome this obstacle we use the concept of weak index pairs.

NAIARA VERGIAN DE PAULO COSTA, Federal University of Santa Catarina
Multiplicity of periodic orbits and homoclinics for Hamiltonian systems in $\mathbb{R}^4$

Due to the conservation of energy, one usually restricts the study of a Hamiltonian system in $\mathbb{R}^4$ to a fixed 3-dimensional energy level. Moreover, the existence of a global surface of section in such a level provides additional reduction of the flow to an area-preserving surface map. In case global sections do not exist or they are unlikely to be found, one may still search for the so called systems of transversal sections. These systems are singular foliations of the energy level so that the singular set is formed by finitely many periodic orbits and the regular leaves are transverse to the Hamiltonian vector field. The Hamiltonian flow may determine transition maps between some regular leaves of a system of transversal sections and valuable information about the dynamics, such as the multiplicity of periodic orbits and homoclinics, may be obtained by means of standard tools in 2-dimensional discrete dynamics. This is a work in progress with Pedro A. S. Salomão (University of São Paulo).

KETTY A. DE REZENDE, University of Campinas
Homological Tools for Understanding Dynamical Systems

We will define a topological context fruitful in obtaining information on the behaviour of a wide range of dynamical systems. We will present an overview of results obtained in the past ten years for Morse-Smale, Morse-Bott, Novikov and Gutierrez-Sotomayor flows. The overarching idea is to define an appropriate filtered chain complex generated by invariant sets. Conley's theory permits to rise above the differentiability requirements of the phase space as well as to consider richer isolated invariant sets. With these tools, we consider as our major algebraic apparatus a spectral sequence of the given filtered chain complex. The unfolding of the spectral sequence exhibits a rich algebraic procedure and provides much insight into dynamical properties (bifurcation, cancellation phenomena etc.) of a continuation of the dynamical systems being studied. Our goal is to present this algebraic-dynamical set-up at an introductory level.

References

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UMBERTO HRYNIEWICZ, Universidade Federal do Rio de Janeiro
Index pairs associated to finite-energy foliations

A finite-energy foliation for a Reeb flow on a 3-manifold, as introduced by Hofer, Wysocki and Zehnder, is a special kind of foliation of its symplectization: leafs are either cylinders over certain periodic orbits, called binding orbits, or project onto transverse Seifert surfaces for some of the binding orbits. In this talk I would like to explain how to find Conley index pairs using such a foliation, and present applications. Typical applications include problems in celestial mechanics, like the Euler problem.

HIROSHI KOKUBU, Kyoto University
Global dynamics of systems with steep nonlinearities

We discusses a novel approach to obtaining mathematically rigorous results on the global dynamics of ordinary differential equations. We study switching models of regulatory networks. To each switching network we associate a Morse graph, a computable object that describes a Morse decomposition of the dynamics. In this talk I show that all smooth perturbations of the switching system share the same Morse graph and we compute explicit bounds on the size of the allowable perturbation. This shows that computationally tractable switching systems can be used to characterize dynamics of smooth systems with steep nonlinearities. This talk is based on a joint work with T. Gedeon (Montanta State), S. Harker (Rutgers), K. Mischaikow (Rutgers), and H. Oka (Ryukoku).

DAHISY LIMA, University of Campinas
Extracting Dynamical Information from a Morse-Novikov Spectral Sequence

The goal of this talk is to obtain dynamical information from algebraic-topological tools found in Conley Theory and used to explore filtered chain complexes and their underlying spectral sequences.

One can describe the qualitative aspects of a Morse-Novikov flow in terms of a chain complex generated by the critical points of a circle-valued Morse function and whose differential counts flow lines (with signs) between them. Endowing this chain complex with an increasing filtration, one can associate to it a spectral sequence (E^r, d^r). We develop an algorithm, Smale's Cancellation Sweeping Algorithm (SCSA), which models the spectral sequence and generates a collection of connection matrices from which one can recover the differentials d^r. Whenever the SCSA identifies a non null differential on the r-th page of (E^r, d^r), one observes algebraic cancellations occurring within the modules E^{r+1}'s.

These algebraic cancellations are dynamically interpreted as cancellations of consecutive critical points. During this process, we keep track of all dynamical information on the birth and death of connecting orbits between critical points, as well as periodic orbits that may arise within a family of circle-valued Morse functions. Furthermore, this family corresponds to a continuation from the initial Morse-Novikov flow to a minimal Morse-Novikov flow.

References

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MARIAN MROZEK, Jagiellonian University in Krakow, Poland
Conley index theory for sampled dynamical systems

In late 90' R. Forman [1, 2] introduced the concept of a combinatorial vector field on a CW complex and presented a version of Morse theory for acyclic combinatorial vector fields. Recently, an extension of this theory towards Conley index theory, has been presented in [3, 4]. In particular, the extension covers such concepts as attractors, repellers, Morse decompositions, Conley-Morse graphs. Moreover, the extension applies to a generalized concept of combinatorial multivector fields. Such fields are better adjusted to the needs of modelling differential equations.

In this talk we will present the foundations of the new theory, the bridges between the classical and combinatorial theory and potential applications to nonlinear differential equations and sampled dynamical systems. In particular, we will show how the combinatorial multivector fields may be used to model the dynamics of a differential equation and how similar methods may be applied to study sampled dynamical systems [5].

References

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HIROE OKA, Ryukoku University
Detecting Morse Decompositions of the Global Attractor of Regulatory Networks by Time Series Data

Complex network structure frequently appear in biological systems such as gene regulatory networks, circadian rhythm models, signal transduction circuits, etc. As a mathematical formulation of such biological complex network systems, Fiedler, Mochizuki and their collaborators (JDDE 2013) recently defined a class of ODEs associated with a finite digraph called aregulatory network, and proved that its dynamics on the global attractor can in principle be faithfully monitored by information from a (potentially much) fewer number of nodes called the feedback vertex set of the graph. In this talk, I will use their theory to give a method for detecting a more detailed information on the dynamics of regulatorynetworks, namely the Morse decomposition of its global attractor. The main idea is to take time series data from the feedback vertex set of a regulatory network, and construct a combinatorial multi-valued map, to which we apply the so-called Conley-Morse Database method. As a test example, we study Mirsky's mathematical model for mammalian circadian rhythm which can be represented as aregulatory network with 21 nodes, and show that numerically generated time series data from its feedback vertex set consisting of 7 nodes correctly detect a Morse decomposition in the global attractor, including 1 stable periodic orbit, 2 unstable periodic orbits, and 1 unstable fixed point. This is a joint work with B. Fielder, A. Mochizuki, G. Kurosawa, and H. Kokubu.

PAWEL PILARCZYK, California State University Channel Islands
Progress in algorithmic computation of the homological Conley index map

Advancement in the development of algorithms and software for automatic analysis of global dynamics in multi-parameter dynamical systems, has recently made it possible to apply the method to a wide class of dynamical models with both discrete and continuous time (the latter through a time-1 map); for example, a disease spreading model in epidemiology (Knipl, Pilarczyk, Röst 2015). One of the most computationally challenging steps is the computation of the Conley index, especially the index map in homology. In this talk, a focus will be on an approach that uses a rectangular grid to construct an index pair and to approximate the map. Recent progress in the computation of the homomorphism induced in homology by a continuous map in this context will be introduced (Harker, Kokubu, Mischaikow, Pilarczyk 2016). This approach uses a combinatorial approximation of the map, and overcomes several constraints that were previously considerably limiting the applicability of this method.

MARIANA SILVEIRA, Federal University of ABC
Bifurcation detected by a Spectral Sequence in the Morse-Smale Setting

The purpose of this talk is present a procedure in the context of Morse-Conley Theory which allows us to obtain dynamical information from algebraic-topological tools.

Given a flow on a compact manifold M and a filtered chain complex whose differential is a connection matrix, we focus on the associated spectral sequence. In this context, we introduce a sweeping algorithm, which codifies in the connection matrix the information of the spectral sequence [1].

The sweeping algorithm produces a family of connection matrices and associated transition matrices which retrieve the information given by the spectral sequence and recover dynamical information of the initial flow. In [1] one shows the existence of paths in the flow associated to nonzero differentials of the spectral sequence.

In this talk we present another result which identifies further dynamical information codified by the sweeping algorithm. Given a Morse-Smale flow in M and its associated family of connection and transition matrices, we introduce in [2] directed graphs, called schematics, which depict the bifurcation that could occur if this sequence of matrices were realized in a flow continuation. In this way, a sequence of schematics can be seen as a continuation where the transition matrices give the information about the bifurcation behavior.

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EWERTON VIEIRA, Universidade Federal de Goiás
Detecting Bifurcations via Transition Matrix

The Conley index theory has been a valuable topological technique for detecting global bifurcations in dynamical systems [1], [2], [3], [4], [5] and [6]. This index is a standard tool in the analysis of invariant sets in dynamical systems, and its significance owes partly to the fact that it is invariant under local perturbation of a flow (the continuation property).

In this setting, we present a new definition and applications of transition matrix as a Conley-index based algebraic transformation that tracks changes in index information under continuation and thereby identifies global bifurcations that could occur during the continuation [7] and [8].

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