Incompressible Fluid Dynamics
Dynamique des fluides incompressibles
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ANNE BRONZI, Universidade Estadual de Campinas
Abstract framework for the theory of statistical solutions

In this talk we will present an abstract framework for the theory of statistical solutions for general evolution equations. This theory extends the notion of statistical solutions initially developed for the 3D incompressible Navier–Stokes equations to other evolution equations that have global solutions which are not known to be unique. The main results are the existence of statistical solution for the initial value problem and the convergence of statistical solutions of regularized equations to statistical solutions of the original one. The wide applicability of the theory will be illustrated with the very 3D incompressible Navier–Stokes equations, a reaction–diffusion equation, a nonlinear wave equation and the 2D inviscid limit of statistical solutions of the Navier-Stokes to the Euler equations.

This is a joint work with Cecilia Mondaini and Ricardo Rosa.

MICHELE COTI-ZELATI, University of Maryland
Mixing and dissipation in two-dimensional fluids

We discuss quantitative properties of mixing and dissipation in two-dimensional fluids at high or infinite Reynolds numbers. The main mechanism to detect is a transfer of information from large length-scales to small length-scales, which, if combined with dissipation, it acts to enhance, in certain senses, the dissipative forces.

TAREK ELGINDI, UC-San Diego
Finite-time singularity formation for De Gregorio's model of the 3d vorticity equation

Abstract: We discuss a proof of singularity formation for strong solutions to De Gregorio's model of the three dimensional vorticity equation. We explain the three main parts of the proof:

(1) Understanding the interaction between vortex stretching and advection and how one could (heuristically) win over the other.

(2) Considering self-similar solutions to the equation without advection and formally perturbing them to get solutions to the (full) equation with advection.

(3) A good understanding of how the Hilbert transform behaves in certain settings.

MILTON C. LOPES FILHO, Universidade Federal do Rio de Janeiro
Newtonian limit of the Euler-alpha model in domains with boundary

We examine recent results concerning the small alpha limit of solutions of the Euler-alpha model in a domain with boundary, both with so slip and with Navier slip boundary conditions.

SUSAN FRIEDLANDER, University of Southern California
A stochastically forced shell model for turbulence

We establish the anomalous mean dissipation rate of energy in the inviscid limit for a stochastic shell model of turbulent fluid flow. The proof relies on viscosity independent bounds for statistically steady states and on establishing ergodic and mixing properties for the viscous model.

This is joint work with Nathan Glatt-Holtz and Vlad Vicol.

NATHAN GLATT-HOLTZ, Tulane University
Stochastic Models for Turbulent Convection

Buoyancy driven convection plays a fundamental role in diverse physical settings: from cloud formation to large scale oceanic and atmospheric circulation processes to the internal dynamics of planets and stars. Typically, such fluid systems are driven by heat fluxes acting both through boundaries (i.e. heating from below) and in the bulk (i.e. internal 'volumetric' heating sources) both of which can have an essentially stochastic nature. In this talk I will discuss some recent mathematical developments concerning ergodicity, singular parameter limits and the onset of instability in the stochastic Boussinesq and Magnetohydrodynamics equations.

SLIM IBRAHIM, University of Victoria
STABILITY OF RECEDING TRAVELING WAVES IN VISCOUS THIN FILMS

We consider the thin-film equation $h_t + (hh_{yyy})_y = 0$ with a zero contact angle at the free boundary, that is, at the triple junction where liquid, gas, and solid meet. Previous results on stability and well-posedness of this equation have focused on perturbations of equilibrium-stationary or self-similar profiles, the latter eventually wetting the whole surface.

In this talk, we consider traveling waves $h = \frac{V}{6}x^3 + \nu x^2$ for $x \geq 0$, where $x = y - Vt$ and $V, \nu \geq 0$ are free parameters. These traveling waves are receding and therefore describe de-wetting, a phenomenon genuinely linked to the fourth-order nature of the thin-film equation and not encountered in the porous-medium case as it violates the comparison principle.

To study the asymptotic stability of these waves, we carry a stability analysis on a linear fourth-order degenerate-parabolic operator for which we prove maximal-regularity estimates to arbitrary orders of the expansion in x in a right-neighborhood of the contact line $x = 0$. This leads to a well-posedness and stability result for the corresponding nonlinear equation. As the linearized evolution scales differently when $x \searrow 0$ and $x \to \infty$, the analysis is more intricate than in related previous works, and offers the opportunity to investigate other situations in which the comparison principle is violated, such as rupture of droplets.

ALEXEI MAILYBAEV, IMPA, Rio de Janeiro
Spontaneously stochastic solutions for the Rayleigh-Taylor instability

The Rayleigh-Taylor instability occurs at an interface between two fluids of different densities, when heavier fluid is placed above the lighter one. Propagation of a disturbance from smaller to larger scales due to nonlinear interaction generates a very complex turbulent dynamics in a growing mixing layer. Occurrence of the RT instability is abundant in nature, which includes astrophysical, geological and atmospheric phenomena, as well as various technological applications such as combustion. In this work we suggest that a turbulent phase of the Rayleigh-Taylor instability can be explained as a universal stochastic wave traveling with constant speed in a properly renormalized system. This wave, originating from ordinary deterministic chaos in a renormalized time, has two constant limiting states at both sides. The theoretical analysis is confirmed with extensive numerical simulations for a novel shell model of convection turbulence.

ANNA MAZZUCATO, Penn State University
The vanishing viscosity limit for an Oseen-type equation

I will discuss the limit of vanishing viscosity for the Navier-Stokes equation linearized around a regular and stationary Euler flow in bounded domains, when the boundary is characteristic for the problem, using an appropriate corrector for the difference between the viscous and inviscid flows. This is joint work with Gung-Min Gie (University of Louisville, USA) and James Kelliher (University of California-Riverside, USA).

DANA MENDELSON, University of Chicago
An infinite sequence of conserved quantities for the cubic Gross-Pitaevskii hierarchy on R

We consider the (de)focusing cubic Gross-Pitaevskii (GP) hierarchy on R, which is an infinite hierarchy of coupled linear non-homogeneous PDE which appears in the derivation of the cubic nonlinear Schrodinger (NLS) equation from quantum many-particle systems. Motivated by the fact that the cubic NLS on R is an integrable equation which admits infinitely many conserved quantities, we exhibit an infinite sequence of operators which generate analogous conserved quantities for the GP hierarchy. This is joint work with Andrea Nahmod, Natasa Pavlovic, and Gigliola Staffilani.

GABRIELA PLANAS, Unicamp

Well-posedness for a non-isothermal flow of two viscous incompressible fluids

This work is concerned with a non-isothermal diffuse-interface model which describes the motion of a mixture of two viscous incompressible fluids. The fluids are assumed to have matched densities and the same viscosity and thermal conductivity. The model consists of modified Navier-Stokes equations coupled with a phase-field equation given by a convective Allen-Cahn equation, and energy transport equation for the temperature. We prove the existence of a global weak solution in two and three dimensions, the existence and uniqueness of the global strong solution in the two-dimensional case, and the local strong solution in the three-dimensional case, without any restriction on the size of the initial data. Joint work with J. Honda Lopes (Unicamp)

ALEXANDER SCHNIRELMAN, COncordia University

On the 2-point problem for the Euler-Lagrange equations

New Abstract: Consider the motion of the ideal incompressible fluid on a compact 2-d manifold M. It is described by the Euler-Lagrange equations. For every initial velocity field v there is a unique 1-parameter family of area-preserving diffeomorphisms $g_t : M \to M$. The time-1 diffeomorphism g_1 is defined by the initial velocity v and is denoted by Exp_v. The map $v \mapsto Exp_v$ is the geodesic exponential map on the group $D(M)$ of area-preserving diffeomorphisms. The main result of the talk is the following

Theorem: for any $ginD$ there exists a vector field v such that $g = Exp_v$.

This theorem looks superficially like the Hopf-Rinow theorem in the finite-dimensional geometry. However, it has little to do with the Hopf-Rinow Theorem, and the proof is based on completely different ideas.

JAVIER GOMEZ SERRANO, Princeton University

Global solutions for the generalized SQG equation

In this talk I will discuss recent results on the existence in different settings of global solutions to the generalized SQG equations, a family of equations that interpolates between the surface quasi-geostrophic equations and the incompressible 2D Euler equations.

SASHA SHNIRELMAN, Concordia University, Montreal, Canada

Constructions of weak solutions of the Euler equations

Weak solutions of the Euler equations are intended to describe turbulent motion of the slightly viscous fluid at very high Raynolds numbers. Important classes of weak solutions were constructed in recent works of C. De Lellis, L. Szekelyhidi and others following the seminal work of V. Scheffer. However, the physical significance of these solutions is not certain because they are too “flexible”. In fact, there are some hidden forces (“ghost forces”) driving these flows. In the present talk I'm going to show some other methods to construct weak solutions which appear to be more physically meaningful.

ANDREI TARFULEA, University of Chicago

Improved estimates for thermal fluid equations

We consider two hydrodynamic model problems (one incompressible and one compressible) with three dimensional fluid flow on the torus and temperature-dependent viscosity and conductivity. The ambient heat for the fluid is transported by the flow and fed by the local energy dissipation, modeling the transfer of kinetic energy into thermal energy through fluid friction. Both the viscosity and conductivity grow with the local temperature. We prove a strong a priori bound on the enstrophy of the velocity weighed against the temperature for initial data of arbitrary size, requiring only that the conductivity be proportionately larger than the viscosity (and, in the incompressible case, a bound on the temperature as a Muckenhoupt weight).