NATE ACKERMANN, Harvard University
Transferring Results from Infinitary Classical Model Theory to Infinitary Continuous Model Theory
In this talk we will provide a method of encoding the category of metric structures (of a given language) via a category whose objects and morphisms are each classical structures (of a closely related language). This encoding will allow us to associate to each infinitary continuous theory a corresponding classical theory, in such a way that a metric structure will satisfy the continuous theory if and only if the encoded structure satisfies the corresponding classical theory. This encoding will then allow us to deduce several results about continuous logic from their classical counterparts. As examples we will be able to deduce for continuous logic a version of Barwise completeness, Barwise compactness, Morley’s two cardinal theorem, deduce the Hanf number for density, as well as make sense of the notion of potentially approximately isomorphic.

XAVIER CAICEDO, Universidad de los Andes, Bogotá
A Lindström’s theorem for continuous logic
Oversimplifying, continuous logic has his prehistory in Chang’s work on Łukasiewicz predicate logic and Chang and Keisler monograph on logic with values in compact Hausdorff spaces. It had an independent revival in Krivine’s successful use of model theoretic methods in Banach spaces, work continued by Henson and iovino and generalized to metric spaces by Ben Yaacov and Usvyatsov. The analogue of Lindström’s characterization of elementary logic in terms of compactness and the downward Löwenheim–Skolem property holds for continuous logic, even if we allow infinitary continuous predicates. The main tools we utilize to prove this maximality result are a natural notion of approximation which behaves well in compact extensions, and a characterization of equivalence of models in continuous logic by means of partial approximations. Some topological ideas are useful also. We will comment other characterizations of continuous logic in the vein of Lindström.

EDUARDO DUEÑEZ, University of Texas at San Antonio
Ergodic theorems and metastability: A continuous logic viewpoint
We revisit the Polynomial Mean-Ergodic Theorem (Poly-MET), which asserts the mean convergence of averages of an abelian unitary polynomial action on a Hilbert space. A special case of Walsh’s theorem, Poly-MET generalizes the classical ergodic theorem of von Neumann. In this talk we focus on actions of the group $\mathbb{Z}, +$. We introduce the class of PET structures (after Bergelson’s technique of ”Polynomial Ergodic-Theoretic (PET)” inductive descent). A PET structure is a Hilbert space $\mathcal{H}$ endowed with a collection $\mathcal{P}$ of polynomial sequences in $U(\mathcal{H})$, plus a few other ingredients (notably, a Følner sequence $\{F_n\}$ and the corresponding averaging operations $AV_n$). The class of all PET structures with a given upper bound $d$ on the degree of the sequences is axiomatizable in a suitable Henson language, so it is an elementary class of Henson structures. By working in saturated PET structures, it becomes possible to formalize (à la Loeb) classical measure-theoretical arguments, leading to a succinct proof of Poly-MET. Exploiting the compactness of Henson’s logic as captured by a ”uniform metastability principle” anticipated by Avigad and iovino, the convergence result admits an immediate refinement to a statement about uniformly metastable convergence. Our approach owes much to Tao’s outline of a nonstandard analysis proof of Walsh’s theorem. Ongoing work formalizes multiple ergodic averages of polynomial actions of nilpotent groups in the framework of Henson structures.

CHRISTOPHER EAGLE, University of Victoria
Topology, abstract model theory, and the omitting types theorem
It has long been known that the Omitting Types Theorem of first-order logic is closely related to the Baire Category Theorem of topology. In joint work with Franklin D. Tall, and building on work of R. Knight, we investigate an abstract framework designed to capture the key topological properties of the family of type spaces associated to a classical logic. In this setting we make the connection between Omitting Types and Baire Category precise for more general logics by showing an equivalence between the Omitting Types Theorem and the Baire property for a particular topological space arising as a limit of type spaces. We also define a game version of omitting types based on the Banach-Mazur game. This leads to the question of whether the game version of omitting types is different from the classical one, which will be discussed in Franklin D. Tall’s talk.

ISAAC GOLDBRING, University of California, Irvine
Boundary amenability of groups via ultrapowers

A group $\Gamma$ is said to act amenably on a compact space $X$ if there is a net of functions $x \mapsto \mu^x_n$ from $\Gamma$ to the set of probability measures on $X$ that is uniformly almost invariant. Thus, in this terminology, a group is amenable if and only if it acts amenably on a one point space. More generally, a group is boundary amenable if it acts amenably on some compact space.

In this talk, we present a novel approach to showing that certain groups are boundary amenable. The approach uses ultrapowers of C*-algebras. We show how this technique can be used to give a new proof of a well-known result, namely that groups that act properly and isometrically on a tree are boundary amenable. We will also mention how this approach might be useful to settle the question of whether or not Thompson’s group is boundary amenable.

This work is joint with Stephen Avsec.

BRADD HART, McMaster University
Practical definability in the model theory of operator algebras

One issue in applied model theory of any kind is the ability to recognize abstractly given sets as definable sets. This problem extends to the recognition of imaginaries as well. Using Beth’s definability and conceptual completeness, I will give examples in the continuous setting of how these tools are used to identify definable sets. In particular, I will give several instances of the use of practical definability in the service of the model theoretic study of operator algebras.

JOSE IOVINO, The University of Texas at San Antonio

MICHAEL MAKKAI, McGill University
A survey of first-order logic with dependent sorts (FOLDS)

FOLDS was introduced in the unpublished monograph I wrote in 1995; see the third item under “Papers” on my website. The last four sections of the fourth item, a paper published in 1998, gives a short introduction without proofs. The syntax of FOLDS is a simplified and generalized version of that of Per Martin-Lof’s type theory. It is parametrized by a general concept of “FOLDS signature”, a slightly more involved version of the Tarskian signature (similarity type) used in model theory. The main novelty is the concept of FOLDS-equivalence, a generalization of the concept of isomorphism. It seems that all concepts of “equivalence” in category theory, including higher categories, are special cases of FOLDS-equivalence. A rough statement of the main meta-theorem, the invariance theorem, proved in the monograph using tools of model theory both for classical and (in a suitable version) for intuitionistic logic, says that the first-order properties of structures of a fixed FOLDS-signature invariant under FOLDS-equivalence are exactly the ones that can be formulated in the FOLDS syntax. There are infinitary generalizations of the invariance theorem. The main applications of FOLDS are to higher-dimensional categories; see the item “The omega-category of all multitopic omega-categories; corrected” on the website. The general model-theory of FOLDS includes an elegant generalization of Per Lindstrom’s classical theorem; it characterizes FOLDS globally in model-theoretical terms that are similar to the ones appearing in Lindstrom’s theorem.
FRANK TALL, University of Toronto

Omitting Types and the Baire Category Theorem

For separable metrizable $X$, the omitting types behaviour of the abstract logic defined from $X$ has a close connection with Baire Category properties of $X$. In joint work with Christopher J. Eagle, we discuss the question of whether the game version of the Omitting Types Theorem for the logic defined from $X$ is strictly stronger than the usual Omitting Types Theorem for this logic. If $X$ is complete, the game version is not stronger, but there is an example distinguishing the two if there is a non-meagre $P$-filter on the natural numbers. Although such filters exist in many models of set theory, it is a longstanding open problem whether they exist in all models. For projective $X$, it follows from a theorem of Medini and Zdomskyy that the Axiom of Projective Determinacy implies the game version is not stronger.

HENRY TOWSNER, University of Pennsylvania

How uniform is provable convergence?

Suppose we prove that some family of sequences always converges. Easy examples show that we cannot generally hope to show that the family shares a uniform rate of convergence. However a variety of results show that, if the family is axiomatized in a suitable theory, the family does have a weaker property, a uniform rate of “metastable convergence”, and further that a computable bound on this rate can be obtained from the proof in a systematic way.

We investigate how to obtain an intermediate property: uniform bounds on the number of times the sequence jumps. We show how to relate bounds on jumps to the behavior of an extension of the sequence to the nonstandard integers, and how to extract computable bounds on the number of jumps from proofs which give strong convergence behavior even in the nonstandard integers. As an application, we obtain uniform bounds on the number of jumps in the so-called nonconventional ergodic averages.

ALESSANDRO VIGNATI, York University

The Model theory of the Jacelon algebra

We study the model theory of the Jacelon algebra $W$, a nonunital C*-algebra that plays the same role as the Jian-Su algebra $Z$ does in the classification theory of amenable C*-algebras. After a brief introduction, we show that $W$ is a Fraïssé limit in the continuous setting of Fraïssé theory as developed by Ben Yaacov and others. From this, we infer few properties of the structure of the algebra itself. This is joint work with B. Jacelon.