Hamiltonian Systems and Celestial Mechanics
Systèmes hamiltoniens et mécanique céleste
(Org: **Florin Diacu** (University of Victoria), **Ernesto Perez-Chavela** (Instituto Tecnológico Autónomo de Mexico) and/et **Cristina Stoica** (Wilfrid Laurier University))

JAIME ANDRADE, Universidad del Bío-Bío
Regularization of the restricted three body problem on surfaces of constant curvature

We consider the restricted three body problem defined on a two-dimensional surface of nonzero constant curvature, which corresponds either to the sphere or the upper sheet of the hyperboloid. In this problem, a collision singularity occurs when the infinitesimal mass particle coincides with the position of one of the primaries. We prove that the singularities due to collision can be locally and globaly regularized through the construction of Levi-Civita and Birkhoff type transformation, respectively. We study some general properties of the Hill regions and we determine sufficient conditions to obtain ejection-collision orbits for the symmetrical problem.

LENNARD BAKKER, Brigham Young University
Topological Existence of Periodic Orbits in a Two-Center Symmetric Pair Problem

The Two-Center Symmetric Pair Problem, derived from the Rhomboidal Symmetric-Mass problem by fixing one symmetric pair, limits, as the mass of the non-fixed symmetric pair goes to zero, to the integrable Euler Two-Center Problem (with the centers having equal mass). Standard KAM Theory indicates that many of the quasi-periodic orbits found in the integrable limit case persist for small values of the mass of the non-fixed symmetric pair. Through topological methods, we investigate the existence of several periodic orbits in the Two-Center Symmetric Pair Problem with and without collisions when the mass of the symmetric pair is not necessarily close to zero.

STEFANELLA BOATTO, Universidade Federal de Rio de Janeiro, Brazil/CentraleSupélec, France
N-body and N-vortex dynamics on surfaces of revolution : curvature and topological contributions

One of the today's challenges is the formulation of the N-body and N-vortex dynamics on Riemann surfaces. In this talk we show how the two problems are strongly related one another when looking at them from the point of view of the intrinsic geometry of the surface where the dynamics takes place. Given a surface M of metric g, the distribution of matter S on M, we deduce the dynamics of the masses and some of its properties. Among other things, we find that in the plane the two masses problem does not obeys to the known Kepler laws. Moreover, Newton's Laws are not longer verified on closed surfaces with variable curvature. For masses on an infinite cylinder we are able to observe topological effects due to the topology of the surface (with D. Dritschel, G. Duarte, R. Schaefer, T. Stuchi).

TOSHIAKI FUJIWARA, Kitasato Univaesity
Figure-eight solution and slalom solutions in function space

The figure-eight solution is a choreographic solution to equal mass planar three-body problem in homogeneous potential $U = \sum 1/r_{ij}^{\alpha}$ for $\alpha > -2$, found by Moore, Chenciner and Montgomery. Choreographic solution is a periodic solution in which three bodies share single closed orbit with equal time delay. Slalom solutions are choreographic solutions that have the same homotopy as k-times repeated figure-eight solution, found by Dmitrašinović and Šuvakov.

We trace the figure-eight solution and slalom solutions with $k = 5$ around $\alpha = 1$ numerically. They are mutually close in function space for some interval of α. The $k = 5$ slalom solutions exist roughly in $0.8 < \alpha < 1.3$. Some end points are pair annihilation/creation of two solutions, and the others are single solutions with two-body collisions. We investigated closely the pair annihilation/creation.

This is a joint work with Hiroshi Fukuda and Hiroshi Ozaki.

MARIAN GIDEA, Yeshiva University

Computer assisted proof of Arnold diffusion in the planar elliptic restricted three-body problem

We consider the planar elliptic restricted three-body problem as a model for Arnold diffusion, with the eccentricity of the elliptical orbits viewed as a perturbation parameter. We show that, for each sufficiently small value of the perturbation, there exist trajectories for which the energy changes by some constant independent of the size of the perturbation. We also show that there are trajectories for which the energy changes chaotically, undergoing symbolic dynamics. These results are obtained via a rigorous computer assisted proof. We provide explicit estimates on the range of perturbation values, and on the diffusion time. This is joint work with Maciej Capinski.

ANTONIO HERNANDEZ-GARDUNO, –

Symmetric bifurcations of relative equilibria and isotropy for an X_2Y molecule

The construction of structure-preserving tubular neighborhoods for Hamiltonian Lie group actions facilitates the study of bifurcating branches of relative equilibria near states with non-trivial isotropy. In this talk we explore how the so-called bundle equations provide a systematic study of this bifurcation problem using the lattice of isotropy subgroups. We illustrate this method discussing persistance and bifurcations of relative equilibria in X_2Y molecules. Here one takes advantage of the fact that the phase space is a cotangent bundle. We will also touch on questions about stability. (Joint work with Miguel Rodríguez-Olmos and Cristina Stoica.)

DING LIANG, Department of Mathematics, Sichuan University, Chengdu 610064, China

Some notes on planar Newtonian four-body central configurations with adjacent equal masses

This paper is concerned with any convex non-collinear central configuration of the planar Newtonian four-body problem with adjacent equal masses, which is an open problem in [2]. In this paper, we prove that if $\angle q_2q_1q_4 = \angle q_1q_2q_3$ or $\angle q_1q_4q_3 = \angle q_2q_3q_4$ holds for four-body convex non-collinear central configuration with adjacent equal masses, then the configuration must be an isosceles trapezoid.

EZEQUIEL MADERNA, Universidad de la República

Applications of weak KAM theory to some gravitational problems

Weak KAM theory has been applied to the Newtonian n body problem. The main result obtained on the dynamics was the abundance of completely parabolic motions. Although this is in itself interesting, we believe that the theory should give more results on the description of the dynamics of this classical problem, as well as for some other gravitational problems. The purpose of this talk will be to describe the state of the art in this research line.

ERNESTO PEREZ-CHAVELA, ITAM (Instituto Tecnológico Autónomo de México)

New families of relative equilibria in the curved N-body problem

The curved N-body problem is a generalization of the classical Newtonian N-body problem to spaces with constant curvature, in this talk we will consider the two dimensional case. Using the cotangent potential as a generalization of the Newtonian one, we describe new families of relative equilibria (an especial kind of periodic orbits which are invariant under the group of isometries acting on the corresponding surface). We show some new families of relative equilibria and study the linear stability of them.

ALESSANDRO PORTALURI, University of Turin

Index theory, Maslov index, Spectral flow, Colliding trajectories, Parabolic motions, Homothetic orbits

We develop an index theory for parabolic and collision solutions to the classical n-body problem and we prove sufficient conditions for the finiteness of the spectral index valid in a large class of trajectories ending with a total collapse or expanding with vanishing limiting velocities. Both problems suffer from a lack of compactness and can be brought in a similar form of a Lagrangian System on the half time line by a regularising change of coordinates which preserve the Lagrangian structure. We then introduce a Maslov-type index which is suitable to capture the asymptotic nature of these trajectories as half-clinic orbits: by taking into account the underlying Hamiltonian structure we define the appropriate notion of geometric index for this class of solutions and we develop the relative index theory. We finally compute the Morse index for a class of collision and parabolic motions in Celestial Mechanics.

TANYA SCHMAH, University of Ottawa
Controlling rigid body attitude via shape change

Satellite attitude control is typically achieved via reaction wheels (i.e. rotors) or magnets, which leave the moment of inertia fixed. We investigate an alternative control mechanism: sliding point masses, which change the moment of inertia and thus the angular velocity, while leaving angular momentum fixed.

Joint work with Cristina Stoica

CRISTINA STOICA, Wilfrid Laurier University
Remarks on the n-body dynamics on surfaces of revolution

We explore the dynamics of n mass points constrained to move on a surface of revolution and with mutual interaction given by some binary potential. We discuss symmetries and determine certain invariant manifolds. We also show that the equivalent of Saari's conjecture fails. Further, we define homographic motions to be those for which the configuration formed by the bodies is planar, orthogonal to the axis of revolution and remains self-similar in the ambient space. For equal masses, using discrete reduction, we show that such motions form a two-degrees of freedom mechanical system with symmetry for which one may provide a complete orbital description. We also comment on the role of Gaussian curvature on the stability of regular n-gon relative equilibria.

ZHIQIANG WANG, Chongqing University, China
central configurations formed by nested regular polygons with unequal masses

It is well known that the central configuration plays an important role in the theory and application of the N-body problem. In this talk, using the properties of circulant matrices, we study some necessary conditions and sufficient conditions for palnar central configurations formed by nested regular polygons with unequal masses.

ZHIFU XIE, The University of Southern Mississippi
Super Central Configurations and the Number of Central Configurations Under Geometric Equivalence

Let the configuration $q = (q_1, q_2, \cdots, q_n)$ be a central configuration of n-body for masses $m = (m_1, m_2, \cdots, m_n)$. The central configuration q for m is called a super central configuration if q is also a central configuration for masses $\tilde{m}$ which is a permutation of m and $m \neq \tilde{m}$. Let p and q be two central configurations for m. Then we call p and q geometrically equivalent if they differ by a rotation followed by a scalar multiplication as well as by a permutation of bodies. In this talk, we discuss how the existence of super central configurations decreases the number of central configurations under geometric equivalence.