Hopf Algebras and Tensor Categories
Algèbres de Hopf et catégories de tenseurs

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COSTEL GABRIEL BONTEA, University of New Hampshire

Classifying pointed braided finite tensor categories

A classical result of Joyal and Street states that pointed braided fusion categories correspond to quadratic forms on abelian groups. In this talk, I will prove a non-semisimple analogue of this result. Namely, that pointed braided finite tensor categories admitting a fiber functor correspond to symmetric bilinear forms on objects in symmetric centers of pointed braided fusion categories. This is a report on a joint work with Dmitri Nikshych.

PAVEL ETINGOF, MIT

A counterexample to the Poincare-Birkhoff-Witt theorem

I will give a counterexample to the Poincare-Birkhoff-Witt theorem in characteristic p (but don't get too excited - in a tensor category that has no realization in vector spaces). Namely, let Ver_p be the Verlinde category over a field k of characteristic $p \geq 5$ (the semisimplification of $\mathrm{Rep}(\mathbb{Z}/p\mathbb{Z}, k)$), and let V be the object of Ver_p corresponding to the 2-dimensional indecomposable representation of $Z/p\mathbb{Z}$. Let $\mathfrak{g} = \mathrm{FLie}_{\leq p}(V)$ be the free Lie algebra of V truncated above degree p. Then $\mathfrak{g}$ fails the PBW theorem; in fact, it is not a Lie subalgebra of any associative algebra. To correct this, we need to add a new axiom to the usual Lie operad axioms in the definition of a Lie algebra. This is already familiar for Lie algebras in characteristic 2, where we need to add the condition that $[x, x] = 0$, and for Lie superalgebras in characteristic 3, where we need to add the condition that $[[x, x], x] = 0$ for odd x. Likewise, in characteristic $p \geq 5$ we need a new axiom which is homogeneous of degree p in a single entry x; I call it the p-Jacobi identity. I will write down this axiom and sketch a proof that together with the Lie operad axioms it suffices for the PBW theorem in Ver_p (I expect that it is so in any symmetric tensor category). For this, I will develop a theory of Koszul duality in symmetric tensor categories.

CÉSAR GALINDO, Universidad de los Andes

Fermionic Modular Categories

A fermion in a braided fusion category is an invertible object such that its generated braided fusion subcategory is equivalent to Svec, the category of super vector spaces. A fermionic modular category (or spin modular category) is a modular category with a fermion. The study of fermionic modular categories has been inspired by fermionic topological phases of matter and spin TQFT's.

The Mueger's centralizer of a fermion in a fermionic modular category is a super-modular category (a braided fusion category with Mueger's center equivalent to Svec). Thus, a fermionic modular category is a braided $\mathbb{Z}/2\mathbb{Z}$-extension of the associated super-modular. We conjecture that every super-modular is included in a fermionic modular category in exactly 16-ways. We refer to this conjecture as the 16-fold way conjecture. The difficulty in resolving the 16-fold way conjecture lies in the existence of at least one minimal extension. We describe fermion in some families of braided fusion categories and analyze explicitly the minimal modular extensions of the super-modular categories using a construction called zesting.

The talk is based on joint work with Paul Bruillard, Tobias Hagge, Siu-Hung Ng, Julia Plavnik, Eric Rowell and Zhenghan Wang.

GASTÓN ANDRÉS GARCÍA, Universidad Nacional de La Plata, Argentina

On the determination of algebraic quantum subgroups

Let G be a connected, simply connected, simple complex algebraic group. The purpose of this talk is to present an strategy to determine all (finite-dimensional) quantum subgroups of a given quantum group associated to G at roots of unity, or in equivalent terms, to determine all Hopf algebra quotients of a certain quantized coordinate algebra of G at roots of unity.

An important problem in the theory of quantum groups is the determination of the general properties that a quantum group should have, since up to date there is no axiomatic definition of an *algebraic* quantum group. In this sense, the description of all possible Hopf algebra quotients of a quantum function algebra, of the known examples would give some insight on the structure of the quantum group. This can be viewed as the quantum version of the classical problem of studying subgroups of a simple algebraic group.

This talk is based on joint work with N. Andruskiewitsch and J. Gutierrez, see [AG], [G] and [GG] for more details.

References

[AG] N. Andruskiewitsch and G. A. Garcia, Quantum Subgroups of a Simple Quantum Group at Roots of One. *Compos. Math.* 145 (2009), 476–500.

[G] G. A. Garcia, Quantum subgroups of $GL_{\alpha,\beta}(n)$. *J. Algebra* 324 (2010), 1392–1428.

[GG] G. A. Garcia and J. Gutierrez, Quantum subgroups of simple twisted quantum groups at roots of one. *Trans. Amer. Math. Soc*, to appear. Preprint: arXiv:1601.00897.

ISTVÁN HECKENBERGER, Philipps University Marburg

Some examples of PBW deformations

In the theory of Nichols algebras many examples of finite-dimensional algebras appear, which currently are mainly studied using Groebner basis techniques. In this talk an alternative approach via PBW deformations is discussed. Although a good understanding of PBW deformations of Koszul algebras is available, the Nichols algebra examples typically do not fall into this theory. Interestingly, standard tools from non-commutative algebra seem to be often very appropriate to study these deformations. The examples under investigation appeared already in work of García Iglesias and Vay. The subject of the talk is based on a joint work with Leandro Vendramin.

VLADISLAV KHARCHENKO, Universidad Nacional Autónoma de México

Free braided nonassociative Hopf algebras and Sabinin τ-algebras

This is a joint work with UALBAI UMIRBAEV (Eurasian National University, Astana, Kazakhstan; Wayne State University, Detroit, USA). Let V be a linear space over a field $\mathbf{k}$ with a braiding $\tau : V \otimes V \to V \otimes V$. We prove that the braiding τ has a unique extension on the free nonassociative algebra $\mathbf{k}\{V\}$ freely generated by V so that $\mathbf{k}\{V\}$ is a braided algebra. Moreover, we prove that the free braided algebra $\mathbf{k}\{V\}$ has a natural structure of a braided nonassociative Hopf algebra (H-bialgebra in sense of Pérez-Izcuierdo) such that every element of the space of generators V is primitive. In the case of involutive braidings, $\tau^2 = \mathrm{id}$, we describe braided analogues of Shestakov-Umirbaev operations and prove that these operations are primitive operations. We introduce a braided version of Sabinin algebras and prove that the set of all primitive elements of a nonassociative τ-algebra is a Sabinin τ-algebra.

MIKHAIL KOTCHETOV, Memorial University of Newfoundland

Applications of affine group schemes to the study of gradings by abelian groups

Recall that an affine group scheme over a field $\mathbb{F}$ is a representable functor from the category of associative commutative unital $\mathbb{F}$-algebras to the category of groups. In view of Yoneda's Lemma, the representing object of such a functor carries the structure of a Hopf algebra and, conversely, every commutative Hopf algebra yields an affine group scheme. Any (naïve) algebraic group can be regarded as an affine group scheme, but not conversely.

It is well known that if A is an $\mathbb{F}$-algebra (not necessarily associative) and G is a group then a G-grading $A = \bigoplus_{g\in G} A_g$ is equivalent to an $\mathbb{F}G$-comodule algebra structure on A. If G is abelian then the latter is equivalent to a homomorphism from the affine group scheme G^D, represented by $\mathbb{F}G$, to the automorphism group scheme $\mathbf{Aut}_{\mathbb{F}}(A)$ (which is representable if $\dim A < \infty$). If $\mathbb{F}$ is algebraically closed of characteristic 0 then it is sufficient to consider the algebraic group $\mathrm{Aut}_{\mathbb{F}}(A)$ (the $\mathbb{F}$-points of the scheme $\mathbf{Aut}_{\mathbb{F}}(A)$), but not in general.

In this talk, we will look at some examples where the above approach allows one to classify all G-gradings on A up to isomorphism.

MITJA MASTNAK, Saint Mary's University
Bialgebras and coverings

Abstract: If A, B are algebras and C a coalgebra, then a linear map $f\colon A \otimes C \to B$ is called a measuring if it corresponds to an algebra map from A to the convolution algebras $Hom(C, B)$, or, intrinsically, if for x, y in A and c in C we have that $f(1_A, c) = \varepsilon(c)1_B$ and $f(xy, c) = f(x, c_1)f(y, c_2)$. If A, B are bialgebras, then we say that f is a partial covering if it is also a coalgebra map. A covering is a surjective partial-covering.

In the talk I will discuss the bi-category of bialgebras, with (partial) coverings and the idea of classifying bialgebras up to covering equivalence. This is joint work with A. Lauve and fits into the general scheme of Grunenfelder and Paré of using coalgebras instead of sets as parameterizing objects.

SONIA NATALE, Facultad de Matemática, Astronomía y Física. Universidad Nacional de Córdoba. CIEM-CONICET.
On the classification of almost square-free modular categories.

This is joint work with Jingcheng Dong. Let $\mathcal{C}$ be a modular category of Frobenius-Perron dimension dq^n, where q is a prime number and d is a square-free integer. We show that if $q > 2$ then $\mathcal{C}$ is integral and nilpotent. In particular, $\mathcal{C}$ is grouptheoretical. In the general case, we describe the structure of $\mathcal{C}$ in terms of equivariantizations of group-crossed braided fusion categories.

CRIS NEGRON, Massachusetts Institute of Technology
Small quantum groups associated to Belavin-Drinfeld triples

For a simple Lie algebra L of type A, D, E, I will explain how any Belavin-Drinfeld triple on the Dynkin diagram of L produces a collection of Drinfeld twists for Lusztig's small quantum group $u_q(L)$. These twists give rise to new finite-dimensional factorizable, ribbon, Hopf algebras. For any Hopf algebra constructed in this manner, I will discuss how one can read off the group of grouplike elements, identify the Drinfeld element, and describe the irreducible representations of the dual from the given Belavin-Drinfeld triple.

VICTOR OSTRIK, University of Oregon
Modular extensions group of a symmetric tensor category.

Group of modular extensions of a symmetric fusion category was introduced by Lan, Kong, and Wen. By definition its elements are modular tensor categories which contain the symmetric fusion category and are of the same size as the Drinfeld center. In this talk I will review known results about this group.

JULIA PLAVNIK, Texas A&M University
On classification of super-modular categories by rank

Super-modular categories are unitary premodular categories whose Mueger center is the category of super-vector spaces sVec generated by a fermion f. These categories are important for both mathematical and physical reasons. For example, they are used to model fermionic topological phases of matter. It is also interesting to pursue a theory of super-modular categories parallel to the one of modular categories. Moreover, the general structure of premodular categories is reduced to that of modular or super-modular categories, which is another motivation to study super-modular categories.

In this talk, we will present basic definitions and properties of super-modular categories and a classification of these categories up to rank 6. This talk is based on a joint work with P. Bruillard, C. Galindo, S-H. Ng, E. Rowell, and Z. Wang.

ERIC ROWELL, Texas A and M University
Metaplectic Modular Categories

I will discuss recent joint work on so-called metaplectic modular categories: slight generalizations of the categories $SO(N)_2$ constructed from quantum groups of types B and D at certain roots of unity. We have a complete classification of these categories and can prove that the braid group representations associated with $SO(N)_2$ have finite image. See ArXiv: 1401.5329 (Quantum Topol. 2017), 1601.05460 (J. Algebra 2016) and 1609.04896 (submitted).

CRISTIAN VAY, CONICET
On projective modules over finite quantum groups

Let $\mathcal{D}$ be the Drinfeld double of the bosonization $\mathfrak{B}(V)\#\Bbbk G$ of a finite-dimensional Nichols algebra $\mathfrak{B}(V)$ over a finite group G. It is known that the simple $\mathcal{D}$-modules are parametrized by the simple modules over $\mathcal{D}(G)$, the Drinfeld double of G. This parametrization can be obtained by considering the head $\mathsf{L}(\lambda)$ of the Verma module $\mathsf{M}(\lambda)$ for every simple $\mathcal{D}(G)$-module λ. We will show that the projective $\mathcal{D}$-modules are filtered by Verma modules and the BGG Reciprocity $[\mathsf{P}(\mu) : \mathsf{M}(\lambda)] = [\mathsf{M}(\lambda) : \mathsf{L}(\mu)]$ holds for the projective cover $\mathsf{P}(\mu)$ of $\mathsf{L}(\mu)$. Analogous results are well-known for highest weight categories. However, the categoy of $\mathcal{D}$-modules is not highest weight.

We shall use graded characters to obtain the BGG Reciprocity as consequence of a graded version of it. As a by-product we will show that a Verma module is simple if and only if it is projective. Also, we will describe the tensor product between projective modules.

MILEN YAKIMOV, Louisiana State University
Prime spectra of 2-categories and categorification of open Richardson varieties

We will describe a general theory of prime, completely prime, semiprime, and primitive ideals of monoidal categories, and more generally of 2-categories. These notions extend Balmer's theory of spectra of tensor triangulated categories, which deals with the symmetric/braided case. The notions provide a bridge between prime spectra of noncommutative rings and total positivity. As an application we obtain categorications of the coordinate rings of Richardson varieties for arbitrary symmetric Kac-Moody algebras. This is a joint work with Kent Vashaw (LSU).

YINHUO ZHANG, University of Hasselt, Belgium
Reconstruction of monoidal categories from their invariants

A monoidal (or tensor) category possess many different invariants; some of them might be decisive and the others might be not. In this talk, we present two decisive invariants of a monoidal category $\mathcal{C}$: the representation (or the Green) ring $r(\mathcal{C})$ and the Auslander algebra $A(\mathcal{C})$. We show that a Krull-Schmidt and abelian monoidal category of finite rank over an algebraically closed field $\mathbb{F}$ can be reconstructed back from its Green ring, the Auslander algebra and the associator.