Harmonic Analysis and Inverse Problems
Analyse harmonique et problèmes inverses

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BEN ADCOCK, Simon Fraser University
Robustness to unknown error in sparse regularization

Sparse regularization has become increasingly popular in the last decade with the advent of techniques such compressed sensing and matrix completion. Most theoretical guarantees for sparse regularization assume that a bound for the noise is known in advance. Yet there are many practical scenarios where such a bound may not be known *a priori*. While estimation of this bound may be possible, e.g. via cross validation, there are few theoretical results in practical settings which explain the effect of such unknown noise on the overall reconstruction. In this talk I will present new results on the performance of several popular sparse regularization techniques under unknown noise. These results cover both Gaussian random matrices, and large classes of structured random matrices corresponding to random sampling with orthonormal systems. Time permitting, I will give several applications of this work, including high-dimensional function approximation, infinite-dimensional sparse regularization for inverse problems, and fast algorithms for non-Cartesian Magnetic Resonance Imaging.

LAURENT BARATCHART,
Inverse potential problems in divergence form on surfaces: non-uniqueness

We consider fields of Newtonian potentials whose defining measure is the divergence of a distribution supported on a surface, the latter being at least Lipschitz smooth. This framework appears in inverse magnetization problems and, more generally, inverse problems arising from Maxwell'sequations in the quasi-static approximation. We describe magnetizations which generate the zero field on one side of the surface, in terms of a spectral integral equation. A technical role is played by a Hardy-Hodge decomposition for vector fields, which is new.

LAURA DE CARLI, Florida International University
p-Riesz bases in quasi shift invariant spaces

Let $\psi \in L^p(\mathbb{R}^d)$, with $1 \le p < \infty$. We investigate Riesz bases in quasi shift invariant space in the form of $V^p(\psi; X) = \overline{\mathrm{Span}\{\tau_{x_j}\psi\}}^{L^p}$ where $X = \{x_j\}_{j\in\mathbb{Z}} \subset \mathbb{R}^d$ is discrete and $\tau_u f(x) = f(x+u)$ is the translation.

MAGALI LOUISE MARIE FOLCH, Universidad Nacional Autónoma de México
Smoothing operators coming from the Navier equation in elasticity

In this talk we will see how two smoothing operators arise from the Navier equation in elasticity and show some of the tools from harmonic analysis used to bound them.

JEAN-PIERRE GABARDO, McMaster University
Weighted Beurling densities and sampling theory

In this talk, we consider Hilbert (and Banach) spaces of functions or distribution supported on a fixed compact subset of $\mathbb{R}^d$ and for which the norm of an element is defined in terms of a weighted L^p-norm of its Fourier transform. The weight in question is assumed to be tempered and moderate. We explore the connection between sampling sets for these spaces and a suitable weighted version of Beurling density. In particular, in the Hilbert space case corresponding to $p = 2$, we obtain weighted versions of the classical density results of H. Landau which relates the measure of a compact set K to the allowable sampling rate for the Fourier transform of the L^2-functions vanishing a.e. outside of K.

GALINA GARCIA, Universidad de Santiago de Chile
A source reconstruction algorithm for the Stokes system from local and missing velocity measurements

We consider the inverse problem of determining the spatial dependence of a source in the Stokes system of the form $f(x)\sigma(t)$ defined in $\Omega \times (0, T)$, assuming that $\sigma(t)$ is known and $f(x)$ is divergence free. The only available observations are single internal measurements of the velocity, in which one of its components is missing. Under some hypothesis on σ we prove uniqueness of this inverse problem via some explicit reconstruction formula. This formula provides the spectral coefficients f_k of the source f in terms of a family of null controls $h^{(\tau)}$ for the corresponding dual system indexed by $\tau \in (0, T]$. We perform numerical simulations in order to illustrate the feasibility, accuracy and stability of the proposed reconstruction formula.

BIN HAN, University of Alberta
Directional Complex Tight Framelets with Applications to Image Processing

Wavelet frames (i.e., framelets) with directionality are of great interest in both theory and application for high dimensional datasets and inverse problems. In this talk we first completely characterize tight framelets. Then we shall introduce a family of directional tensor product complex tight framelets (TPCTFs). The TPCTFs have all the desired properties of both classical wavelets and the discrete cosine transform. The tensor product structure of TPCTFs also means simple computationally fast algorithms for high dimensional problems. For several inverse problems in image processing such as image/video denoising and inpainting, we shall show that TPCTFs have impressive performance over many other known transform-based methods such as curvelets, shearlets, dual tree complex wavelet transform, and undecimated wavelet transform. Moreover, such TPCTFs can be made spatially compactly supported and can be employed in many other applications. This talk is based on several joint papers with Q. Mo, Z. Zhao, and X. Zhuang.

PILAR HERREROS, P. Universidad Católica de Chile
Boundary Rigidity with nonpositive curvature

In general, boundary rigidity refers to the question of whether the metric on a manifold is determined by some data on the boundary. We will focus on results involving the scattering data of the region; for each point and inward direction on the boundary, it associates the exit point and direction of the corresponding unit speed geodesic. We will discus conditions on spaces of nonpositive curvature where we have scattering rigidity or lens rigidity, where the boundary data considered is the scattering data plus the length of each geodesic.

MATTHEW HIRN, Michigan State University
Learning many body physics via wavelet scattering transforms

Deep learning algorithms are making their mark in machine learning, obtaining state of the art results across computer vision, natural language processing, auditory signal processing and more. A wavelet scattering transform has the general architecture of a convolutional neural network, but leverages structure within data by encoding multiscale, stable invariants relevant to the task at hand. This approach is particularly relevant to data generated by physical systems, as such data must respect underlying physical laws. We illustrate this point through many body physics, in which scattering transforms can be loosely interpreted as the machine learning version of a fast multipole method (FMM). Unlike FMMs, which efficiently simulate the physical system, the scattering transform learns the underlying physical kernel from given states of the system. The resulting learning algorithm obtains state of the art numerical results for the regression of molecular energies in quantum chemistry, obtaining errors on the order of more costly quantum mechanical approaches.

ZUHAIR NASHED, University of Central Florida
Perturbation Theory of Generalized Inverse Operators and Ill-Posed Inverse Problems

A perturbation theory of generalized inverses of bounded linear operators will be presented from the perspective of regularizers of ill-posed linear operator equations. This leads to a new approach of using outer inverses for regularizing ill-posed linear operator equations in Hilbert and Banach spaces. For nonlinear operator equations or for least squares nonlinear problems, the approach provides an effective stable substitute for the inverse or generalized inverse of the derivative of the nonlinear operator.

KASSO A. OKOUDJOU, University of Maryland
The HRT Conjecture for real-valued functions

Given a non-zero square integrable function g and $\Lambda = \{(a_k, b_k)\}_{k=1}^{N} \subset \mathbb{R}^2$ let $\mathcal{G}(g, \Lambda) = \{e^{2\pi i b_k \cdot} g(\cdot - a_k)\}_{k=1}^{N}$. The Heil-Ramanathan-Topiwala (HRT) Conjecture is the question of whether $\mathcal{G}(g, \Lambda)$ is linearly independent. For the last two decades, very little progress has been made in settling the conjecture. In this talk, I will introduce an extension principle to investigate the HRT conjecture. More specifically, knowing that the conjecture holds for a given $g \in L^2(\mathbb{R})$ and a given set $\Lambda = \{(a_k, b_k)\}_{k=1}^{N} \subset \mathbb{R}^2$ I will characterize the set of all points $(a, b) \in \mathbb{R}^2 \setminus \Lambda$ such that the conjecture remains true for the same function g and the new set of point $\Lambda_1 = \Lambda \cup \{(a, b)\}$. I will illustrate this for the cases $N = 3$ and 5 and when g is a real-valued function.

QIYU SUN, University of Central Florida
Phaseless sampling and reconstruction of real-valued signals in shift-invariant spaces

Sampling in shift-invariant spaces is a realistic model for signals with smooth spectrum. The topics of this talk are phaseless sampling and reconstruction of real-valued signals in a shift-invariant space from their magnitude measurements on the whole Euclidean space and from their phaseless samples taken on a discrete set with finite sampling density. In this talk, we introduce an undirected graph to a signal and use connectivity of the graph to characterize whether the signal can be determined, up to a sign, from its magnitude measurements on the whole Euclidean space. In this talk, we also consider reconstruction algorithms which provides a suboptimal approximation to the original signal when its noisy phaseless samples are available only.

SUI TANG, Johns Hopkins University
Phase retrieval of evolving signals from space-time samples

Assume that $f \in \mathbb{R}^n$ is an unknown function evolving under the action of an operator A on $\mathbb{R}^n$ such that at time n the signal evolves to $f_n = A^n f$. Let $\Omega \subset \{1, 2, \cdots, n\}$. We consider the problem of finding conditions on A, Ω and L_i such that any $f \in \mathbb{R}^n$ can be uniquely determined up to a sign from the unsigned samples

$$Y = \{|f(i)|, \cdots, |A^{L_i - 1} f(i)| : i \in \Omega\}.$$

This talk is based on the joint work with Akram Aldroubi and Ilya Krishtal.

GIANG TRAN, University of Texas at Austin and University of Waterloo
Sparse Optimization in Learning Governing Equations for Time Varying Measurements

Learning the governing equations for time-varying measurement data is of great interest across different scientific fields. Recovering the governing equations becomes quite challenging, when such data is moreover highly corrupted or does not hold certain properties. In this work, we show that if the data or one data set exhibits chaotic behavior, it is possible to recover the underlying governing nonlinear differential equations by solving an l1 minimization problem which assumes a parsimonious representation of the system and exploit the joint sparsity in different context. Theoretical reconstruction guarantees are obtained by combining recent results on statistical properties of chaotic data with results from compressed sensing theory. Numerical methods are based on iterative thresholding operators and various numerical examples including the Lorenz equations and high dimensional ODEs will be presented to illustrate the power, generality, and efficiency of our model.

AHMAD ZAYED, DePaul University
A New Two-Dimensional Fractional Fourier Transform and the Wigner Distribution

The fractional Fourier transform $F_\theta(w)$ with an angle θ of a function $f(t)$ is a generalization of the standard Fourier transform and reduces to it when $\theta = \pi/2$. It has many applications in signal processing and optics because of its close relations with a number of time-frequency representations. It is known that the Wigner distribution of the fractional Fourier transform $F_\theta(w)$ may be obtained from the Wigner distribution of f by a two-dimensional rotation with the angle θ in the $t - w$ plane

The fractional Fourier transform has been extended to higher dimensions by taking the tensor product of one-dimensional transforms; hence, resulting in a transform in several but separable variables.

The aim of this talk is two-fold: 1) To introduce a new definition of the two-dimensional fractional Fourier transform that is not a tensor product of two copies of one-dimensional transforms. 2) To show that the Wigner distribution of the new two-dimensional fractional Fourier transform $F_{\theta,\phi}(v, w)$ may be obtained from the Wigner distribution of $f(x, y)$ by a more genuine and general four-dimensional rotation with angles $(\theta + \phi)/2$ and $(\theta - \phi)/2$ in two planes of rotations.