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**Gauge Theory and Special Geometry**  
**Théorie de jauge et géométrie spéciale**

(Org: **Benoit Charbonneau** (University of Waterloo), **Sergey Cherkis** (University of Arizona) and/et **Marcos Jardim** (UNICAMP))

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**ANDREW CLARKE**, Universidade Federal do Rio de Janeiro

*$G_2$  structures and the Strominger system in dimension 7*

We consider  $G_2$  structures with torsion coupled with  $G_2$ -instantons, on a compact 7-dimensional manifold. The coupling is via an equation for 4-forms which appears in supergravity and generalized geometry, known as the Bianchi identity. The resulting system of partial differential equations can be regarded as an analogue of the Strominger system in 7-dimensions. We initiate the study of the moduli space of solutions and show that it is finite dimensional using elliptic operator theory.

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**HENRIQUE SÁ EARP**, Unicamp, Brazil

*Construction of  $G_2$ -instantons via twisted connected sums*

We propose a method to construct  $G_2$ -instantons over a compact twisted connected sum  $G_2$ -manifold, applying a previous gluing theorem to instantons over a pair of 7-manifolds with a tubular end. In our example, the moduli spaces of the ingredient instantons are non-trivial, and their images in the moduli space over the asymptotic cross-section K3 surface intersect transversely. Such a pair of asymptotically stable holomorphic bundles is obtained using a twisted version of the Hartshorne-Serre construction, which can be used to produce many more examples. Moreover, their deformation theory and asymptotic behaviour are explicitly understood, results which may be of independent interest. This is joint work with G. Menet and J. Nordström, and it builds on previous works with T. Walpuski and also M. Jardim, G. Menet and D. Prata.

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**LORENZO FOSCOLO**, Stony Brook University

*Non-compact  $G_2$  manifolds from asymptotically conical Calabi-Yau 3-folds*

$G_2$  manifolds are the Riemannian 7-manifolds with  $G_2$  holonomy. Every  $G_2$  manifold is necessarily Ricci-flat. Only four examples of complete non-compact  $G_2$  manifolds are currently known. In joint work with Mark Haskins and Johannes Nordström we construct infinitely many families of new complete non-compact  $G_2$ -holonomy manifolds. The underlying smooth 7-manifolds are all circle bundles over asymptotically conical (AC) Calabi-Yau manifolds of complex dimension 3. The metrics are circle-invariant and their geometry at infinity is that of a circle bundle over a Calabi-Yau cone with fibres of fixed finite length. The  $G_2$  manifolds we construct are therefore 7-dimensional analogues of 4-dimensional ALF hyperkähler metrics.

The dimensional reduction of the equations for  $G_2$  holonomy in the presence of a Killing field was considered by Apostolov-Salamon and by several groups of physicists. We reinterpret the dimensionally-reduced equations in terms of a pair consisting of an  $SU(3)$  structure on the 6-dimensional orbit space coupled to an abelian Calabi-Yau monopole on this 6-manifold. We solve this coupled system of non-linear PDEs by considering the adiabatic limit in which the circle fibres of the associated circle-invariant  $G_2$ -holonomy metrics collapse. The  $G_2$ -holonomy metrics we construct should be thought of as arising from abelian Hermitian-Yang-Mills connections on AC Calabi-Yau 3-folds, especially AC Calabi-Yau metrics on crepant resolutions of Calabi-Yau cones. All our examples provide instances of families of  $G_2$ -holonomy metrics that collapse with bounded curvature to Calabi-Yau 3-folds. This collapse with globally bounded curvature is a new feature of  $G_2$ -holonomy metrics compared to the 4-dimensional ALF hyperkähler setting.

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**ANDRIY HAYDYS**, Albert Ludwigs University of Freiburg

*The Seiberg-Witten equations with multiple spinors in dimension three*

I will discuss some properties of the moduli space of solutions of the Seiberg-Witten equations with multiple spinors in dimension three. Time permitting, I will also touch on relations of these equations with other gauge-theoretic problems.

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**PEDRAM HEKMATI**, University of Auckland  
*E-polynomials of singular character varieties*

A nice application of the Weil conjectures is to compute the Betti numbers of smooth complex projective varieties by counting points over a finite field. For singular or non-compact varieties one is lead to consider the "virtual Hodge numbers" encoded by the E-polynomial, a refinement of the topological Euler characteristic. We will review the arithmetic approach to computing the E-polynomial and discuss the calculation for certain singular character varieties (i.e. moduli spaces of flat connections). This is joint work with David Baraglia.

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**LJUDMILA KAMENOVA**, Stony Brook University  
*Hyperbolicity in hyperkaehler geometry*

The Kobayashi pseudometric on a complex manifold  $M$  is the maximal pseudometric such that any holomorphic map from the Poincare disk to  $M$  is distance-decreasing. Kobayashi conjectured that this pseudometric vanishes on Calabi-Yau manifolds, and in particular, Calabi-Yau manifolds are Kobayashi non-hyperbolic. Using ergodicity of complex structures, together with S. Lu and M. Verbitsky we prove this conjecture for all K3 surfaces and for most classes of hyperkaehler manifolds. In the talk I will also give the algebraic version of hyperbolicity. Together with M. Verbitsky we prove that projective hyperkaehler manifolds with Picard rank at least two are algebraically non-hyperbolic.

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**ANDRES LARRAIN-HUBACH**, University of Dayton  
*The Nahm Transform on Taub-NUT Space*

This talk is based on joint work with Sergey Cherkis and Mark Stern. The Nahm transform is a correspondence between the moduli space of Anti-Self Dual connections with square integrable curvature, also called instantons, and other moduli spaces constructed using representation theoretic information. In the case of the Taub-NUT space, generic instantons correspond to representations of Cherkis bows. The purpose of this talk is to give an outline of this correspondence, focusing on explaining how to construct a bow representation from an instanton on Taub-NUT space.

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**RUXANDRA MORARU**, University of Waterloo  
*Hermitian-Einstein equations on generalized Kähler manifolds*

In this talk, we discuss an analogue of the Hermitian-Einstein equations for generalized Kähler manifolds. We explain in particular how these equations are equivalent to a notion of stability for generalized holomorphic bundles, and that there is a Kobayahsi-Hitchin-type correspondence between solutions of these equations and stable bundles. We also describe moduli spaces of these stable generalized holomorphic bundles on some specific examples of generalized Kähler manifolds.

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**ÁKOS NAGY**, University of Waterloo / Fields Institute  
*The Berry Connection of the Ginzburg–Landau Vortices*

In this talk, I will analyze the vortex equations in dimension 2, and establish asymptotic formulas for the tangent vectors of the vortex moduli space. As an application, I will compute the corresponding Berry phases in the large area limit.

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**GONÇALO OLIVEIRA**, Duke University and IMPA  
*Gauge Theory and  $SU(3)$  structures*

I will report on joint work with Gavin Ball on some gauge theoretical equations that can be written in the presence of an  $SU(3)$ -structure. These can be thought of as generalizing the notion of holomorphic and Hermitian-Yang-Mills equations in the case when the underlying complex structure is not integrable.

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**ANDREW B. ROYSTON**, Texas A&M University  
*Magnetic Monopoles and  $N=2$  super Yang-Mills*

We translate recent developments in quantum Yang-Mills theory with  $N = 2$  supersymmetry into statements about the kernel of certain Dirac operators, or the cohomology of certain Dolbeault operators, over monopole moduli space. This leads to a generalization of Sen's conjecture concerning the nature of the Dolbeault cohomology, and to predictions for when the Dirac operators fail to be Fredholm. This talk is based on work done in collaboration with Daniel Brennan, Greg Moore, and Dieter Van den Bleeken.

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**FLORENT SCHAFFHAUSER**, Universidad de Los Andes  
*Hitchin components for fundamental groups of 2-orbifolds*

Let  $Y$  be a compact connected 2-orbifold of negative Euler characteristic and let  $\pi$  be its orbifold fundamental group. For  $n > 1$ , we denote by  $\mathcal{R}(\pi, n)$  the space of representations of  $\pi$  into  $\mathbf{PGL}(n, \mathbb{R})$ . The purpose of the talk is to show that  $\mathcal{R}(\pi, n)$  possesses connected components homeomorphic to an open ball whose dimension we compute explicitly (for  $n = 2$  and 3, we find again formulae due to Thurston and to Choi and Goldman, respectively). We then give applications of the result to the study of rigidity properties of hyperbolic Coxeter groups. This is joint work with Daniele Alessandrini and Gye-Seon Lee (University of Heidelberg).

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**LAURA SCHAPOSNIK**, University of Illinois at Chicago  
*On some singular fibres of the Hitchin fibration*

The Hitchin fibration is a natural tool through which one can study the moduli space of Higgs bundles and its interesting subspaces (branes). We shall dedicate this talk to the study of certain singular fibres of Hitchin fibrations, obtain correspondences between fibres, and provide a geometric description of branes which lie entirely over the singular loci.

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**MARK STERN**, Duke University  
*Instantons on ALF spaces*

This talk is based on joint work with Sergey Cherkis and Andres Larrain-Hubach. I will discuss progress on establishing Cherkis's Nahm transform for multicenter Taub NUT spaces.