Geometry of Differential Equations, Real and Complex
Géométrie des équations différentielles, réelles et complexes
(Org: **Ronaldo Alves Garcia** (Federal University of Goiás), **Mikhail Malakhaltsev** (University of los Andes), **Daniel Offin** (Queens University), **Jesus Mucino Raymundo** (National Autonomous University of Mexico) and/et **Farid Tari** (University of São Paulo))

MARTHA P. DUSSAN ANGULO, University of Sao Paulo. Brazil
Bjorling problem for timelike surfaces and solutions of homogeneous wave equation

In this talk we show how to construct split-holomorphic extensions of initial curves $\gamma(t)$ in the Lorentz space $\mathbb{R}^3_1$, using, in a natural way, the point of view of solutions of homogeneous wave equation. After extending that curve to a subset of split-complex plane, we solve explicitly the Bjorling problem for timelike surfaces in the Lorentzian spaces $\mathbb{R}^3_1$, $\mathbb{R}^4_1$ and $\mathbb{R}^4_2$. As consequences we construct new examples and give applications. In particular, we describe one-parameter families of timelike surfaces which are solutions of the timelike Bjorling problem. In addition, we also establish symmetry principles for the class of minimal timelike surfaces in those ambient spaces. These results are part from the published papers by the author in Journal of Mathematical Analysis and Applications, Journal of Geometry and Physics and Annali di Matematica Pura ed Applicata.

We remember that the classical Bjorling problem was proposed by Bjorling in 1844 and consists of the construction of a minimal surface in $\mathbb{R}^3$ containing the strip in the interior. The solution was obtained by Schwarz in 1890 through of a explicit formula in terms of initial datas. After that, the Bjorling problem has been considered in other ambient spaces, including in bigger codimmension or with indefinite metrics.

ALEXANDER CARDONA, Universidad de los Andes
Spectral invariants and global pseudo-differential calculus on homogeneous spaces

Global pseudo-differential calculus on compact Lie groups and homogeneous spaces gives, via representation theory, a semi-discrete description of the global analysis and spectral theory of a wide class of operators on these objects. Based on the theory developed by Ruzhansky and Turunen, during this talk we will consider spectral invariants of index type for global homogeneous pseudo-differential operators; some examples and potential applications will be addressed.

JEAN CARLOS CORTISSOZ, Universidad de los Andes
On Bloch's Theorem

A classical theorem of André Bloch guarantees that there is a $B > 0$ such that for any holomorphic function $f : D \longrightarrow \mathbb{C}$, where $D \subset \mathbb{C}$ is the unit disk, such that $|f'(0)| = 1$, there is a subdomain $D' \subset D$, so that f restricted to D' is one to one and $f(D')$ contains a disk of radius B. Computing the optimal value of B is an open problem. In this talk, we will discuss a new proof of Bloch's theorem, and a possible approach to improve on the known estimates on B. This is joint work with Julio Montero.

DÉBORA LOPES DA SILVA, Universidade Federal de Sergipe
Codimension one partially umbilic singularities of hypersurfaces of $\mathbb{R}^4$

This talk is about the mutually orthogonal one dimensional singular foliations, in oriented three dimensional manifolds $\mathbb{M}^3$, whose leaves are the integral curves of the principal curvature direction fields associated to immersions $\alpha : \mathbb{M}^3 \to \mathbb{R}^4$. We focus on behavior of these foliations around singularities defined by the points, called partially umbilic, where at least two principal curvature coincide. It will be described the generic behavior of the foliations in the neighborhood of partially umbilic points of codimension one. These are the singularities which appear generically in one parameter families of hypersurfaces. We express the codimension one condition by minimally weakening the genericity condition given by R. Garcia, D. Lopes e J.

Sotomayor in *Partially Umbilic Singularities of Hypersurfaces of $\mathbb{R}^4$. Bulletin des Sciences Mathematiques (Paris. 1885), v. 139, p. 431-472, (2015).*

RONALDO ALVES GARCIA, Universidade Federal de Goiás
Darboux curves on surfaces

In 1872, Gaston Darboux defined a family of curves on surfaces in the 3-dimensional Euclidean space $\mathbb{R}^3$ which are preserved by the action of the Möbius group and share many properties with geodesics. In this talk the Darboux curves will be considered under a dynamical viewpoint and described globally in special canal surfaces, quadrics and some Darboux cyclides. It will be based, mainly on the paper by R. Garcia; R. Langevin; P. Walczak, Darboux curves on surfaces II. Bull. Braz. Math. Soc. (N.S.) 47 (2016). Some open problems will be posed.

NABIL KAHOUADJI, Northeastern Illinois University
Isometric Immersions of Pseudo-Spherical Surfaces via Differential Equations

Pseudo-spherical surfaces are surfaces of constant negative Gaussian curvature. A way of realizing such a surface in 3d space as a surface of revolution is obtained by rotating the graph of a curve called tractrix around the z-axis (infinite funnel). There is a remarkable connection between the solutions of the sine-Gordon equation $u_{xt} = \sin u$ and pseudo-spherical surfaces, in the sense that every generic solution of this equation can be shown to give rise to a pseudo-spherical surface. Furthermore, the sine-Gordon equation has the property that the way in which the pseudo-spherical surfaces corresponding to its solutions are realized geometrically in 3d space is given in closed form through some remarkable explicit formulas. The sine-Gordon equation is but one member of a very large class of differential equations whose solutions likewise define pseudo-spherical surfaces. These were defined and classified by Chern, Tenenblat and others, and include almost all the known examples of "integrable" partial differential equations. This raises the question of whether the other equations enjoy the same remarkable property as the sine-Gordon equation when it comes to the realization of the corresponding surfaces in 3d space.

We will see that the answer is no, and will provide a full classification of second-order hyperbolic and kth-order evolution equations. The classification results will show, among other things, that the sine-Gordon equation is quite unique in this regard amongst all integrable equations.

MIKHAIL MALAKHALTSEV, Universidad de los Andes, Bogotá, Colombia
Binary differential equations and 3-webs with singularities

A 3-web with singularities is a geometric structure locally given by three one-dimensional distributions on an open dense subset U of a two-dimensional manifold M. A point in U is called *regular* if values of the distributions are pairwise transversal at this point, all the other points of M are called *singular*.

A binary differential equation of third degree determines a 3-web with singularities (see, for example, T. Fukui and J. J. Nuño-Ballesteros, Isolated singularities of binary differential equations of degree n, Publicacions Matemátiques, vol. 56, 65–89, 2012). We describe singularities of this 3-web, and show how to find topological and differential invariants of these singularities using methods developed in the paper F.A. Arias, J.R. Arteaga, and M. Malakhaltsev, 3-webs with singularities, Lobachevskii J. of Math, 37 (1), 1–20, 2016.

KENNETH MEYER, University of Cinncinnati
Asymptotic Stability Estimates near an Equilibrium Point

We use the error bounds for adiabatic invariants found in the work of Chartier, Murua and Sanz-Serna to bound the solutions of a Hamiltonian system near an equilibrium over exponentially long times. Our estimates depend only on the linearized system and not on the higher order terms as in KAM theory, nor do we require any steepness or convexity conditions as in Nekhoroshev theory. We require that the equilibrium point where our estimate applies satisfy a type of formal stability called Lie stability.

DANIEL OFFIN, Queen's University
Multiple periodic solutions in classical Hamiltonian systems.

The question of multiple periodic solutions on energy surfaces, has a long and distinguished history. We will consider the question of multiple periodic solutions on non compact energy surfaces, and find conditions which guarantee infinitely many. We use variational techniques including the mountain pass theorem, and a result of the author which guarantees that a minimizing periodic solution must be hyperbolic on its energy surface.

REGILENE DELAZARI DOS SANTOS OLIVEIRA, ICMC-Universidade de São Paulo
Singular levels and topological invariants of Morse Bott integrable systems on surfaces

In this talk we shall classify (up to homeomorphisms) closed curves and eights of saddle points on orientable closed surfaces. This classification is applied to Morse Bott foliations and Morse Bott integrable systems to define a complete invariant. We also present a realization Theorem based in two transformations and one generator.

These results are part of a joint work with José Martínez-Alfaro (UV, Spain) and Ingrid Sarmiento-Mesa (IBILCE-UNESP, Brazil).

ALESSANDRO PORTALURI, University of Turin
Index and stability of closed semi-Riemannian geodesics

A celebrated result of Poincaré asserts that a closed minimizing geodesic on a orientable (Riemannian) surface is unstable when considered as an orbit of the geodesic flow.

In this talk starting from this classical result, we'll discuss some recently results on the strong and linear instability of closed geodesics of any causal character on higher dimensional (maybe not oriented) Lorentzian and more general semi-Riemannian manifolds.

Dropping the non-positivity assumption of the metric tensor is a quite challenging task since the Morse index is truly infinite.

This is a joint work with X. Hu and R. Yang

JESUS MUCINO RAYMUNDO, Centro de Ciencias Matemáticas, Campus Morelia, Universidad Nacional Autónoma de México
Essential singularities of complex analytic vector fields on $\mathbb{C}$

Let X be a complex analytic vector field on $(\mathbb{C}, 0)$. The real trajectories of it are geodesics of a suitable singular flat metric. Analogously to the classical Picard's Theorem, in the vicinity of an isolated essential singularity, the local complexity of X must be studied using certain "global" flow box maps. We describe the geometry encoded is the simplest cases, for these kind of singularities.

Joint work with A. Alvarez–Parrilla.

ANA RECHTMAN, Universidad Nacional Autonoma de Mexico
The trunkennes of a flow

Motivated by an invariant for knots known as the trunk, we define the trunkennes of a vector field. This quantity is invariant under mesure preserving homeomorphisms and is not proportional to helicity, as almost all known asymptotic invariants. I will explain the construction and the main results related to this invariant. This is joint work with Pierre Dehornoy.

JOHANNA GARCIA SALDAÑA, Catholic University of the Most Holy Conception, Chile
An approach to the period function through the harmonic balance

Each differential system with a period annulus P has associated a period function T. The geometry of T is determined by the number and properties of its critical periods, i.e. its critical points. The critical periods in T is a counterpart to the problem of limit cycles in polynomial systems, since they play a fundamental roll in the geometry of the phase portrait of the differential system. In fact, several analytic techniques have been developed for studying these problems. In this talk we will show that the shape of T can be recovered by using a quantitative approach: the harmonic balance.

ADOLFO GUILLOT SANTIAGO, Universidad Nacional Autónoma de México
Algebraic differential equations with uniform solutions

In the complex domain, the solutions of an ordinary differential equation may present multivaluedness. We will talk about a recent result describing the ordinary differential equations (given by rational vector fields on complex algebraic surfaces) that have a solution that is not multivalued.

FREDERICO XAVIER, Texas Christian University
On the inversion of real polynomial maps

There is some evidence to support the conjecture that a polynomial local diffeomorphism of $\mathbb{R}^n$ into itself, $n \geq 3$, is injective if the pre-images of all 2-planes in $\mathbb{R}^n$ are homeomorphic to connected subsets of $\mathbb{R}^2$. In this talk, we discuss this problem and offer proofs of some related global invertibility results. The arguments involve geometric constructions that use arguments from topology and complex analysis. Part of this work is joint with S. Nollet.