Geometry and Combinatorics of Cell Complexes
Géométrie et combinatoire des complexes cellulaires
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ANDRÉS ANGEL, Universidad de los Andes
Evasiveness of graph properties and graphs on 2p vertices.

A property of graphs on n vertices is said to be evasive if its query complexity is the maximum $n(n-1)/2$. The evasiveness conjecture for graph properties asserts that every non-trivial monotone graph property is evasive.

Kahn, Saks and Sturtevant proved the validity of conjecture when the number of vertices n is a prime power. They also prove the 6 vertices case. It remains open in all other cases, even for $n = 10$ vertices.

For graphs on $2p$ vertices, where p is prime, we give estimations of the Euler characteristic of simplicial complexes associated to potential non-evasive monotone graph properties. Finally we test our estimations in the cases of 10 vertices.

LOUIS BILLERA, Cornell University
On the real linear algebra of binary vectors

We are interested in understanding the real linear relations among all 0-1 vectors in $\mathbb{R}^n$, i.e., the linear matroid over $\mathbb{R}$ on the set of $2^n - 1$ nonzero n-vectors whose coordinates are 0 or 1. This fundamental combinatorial object is at the root of questions that have arisen in a variety of fields, from economics to circuit theory to quantum physics, over the past 50 years, a period spanning the development of modern enumerative combinatorics. Yet there has been little real progress in understanding some of the most basic questions here.

In particular, in many applications it is of interest to know the number of regions in $\mathbb{R}^n$ determined by the $2^n - 1$ linear hyperplanes having 0-1 normals. This number can be obtained from the characteristic polynomial of the geometric lattice of all subspaces in $\mathbb{R}^n$ spanned by these 0-1 vectors. These characteristic polynomials are known only through $n = 7$, while just the number of regions is known for $n = 8$.

We discuss the various contexts in which this question has arisen and describe various general approaches to compute the characteristic polynomial, the most promising at the moment being counting the faces of the corresponding broken circuit complex. This is helped by the fact that while this matroid is not truly "binary" (i.e., representable over the field of 2 elements), it is close (it has a binary matroid as a weak image). Along these lines, we give some very partial results toward a general solution.

MICHAEL DAVIS, Ohio State University
The Euler Characteristic Conjecture and the Charney-Davis Conjecture

The Euler Characteristic Conjecture is the following assertion about the sign of the Euler characteristic of any closed aspherical manifold M^{2k}: $(-1)^k M^{2k} \geq 0$. In the case of higher dimensional Haken manifolds it is closely related to the Charney-Davis Conjecture on the sign of a quantity attached to flag triangulations of odd dimensional spheres. I will survey work on these conjectures and related topics.

EMANUELE DELUCCHI, Université de Fribourg (Switzerland)
Combinatorial models for toric arrangements

The study of the topology of arrangements in the complex torus can be seen as a step beyond the nowadays classical theory of hyperplane arrangements and has been in the focus of a significant amount of recent research. Aftergiving a precise definition, I will illustrate the state of the art on this subject with a special focus on the advances where the use of combinatorial cellular models has proved to be an essential tool. As time will permit, I will also outline some open questions.

ANTON DOCHTERMANN, Texas State University
Co-parking functions and h-vectors of graphical matroids

The h-vector of a simplicial complex X is a well-studied invariant keeping track of the number of i-dimensional faces of X. When X is the independence complex of a matroid Stanley has conjectured that the h-vector is a 'pure O-sequence', i.e. the degree sequence of a monomial ideal generated in a single degree. The conjecture has inspired a good deal of research and is proven in some important cases (but open in general). Merino has established the conjecture for the case that X is a cographical matroid by relating the h-vector to properties of chip-firing on the underlying graph (via a Tutte polynomial evaluation). This approach has been further refined by studying bijections between the set of 'G-parking functions' and the set of spanning trees of G which preserve desired degree/inversion statistics. We introduce and study the notion of a 'coparking' function on a graph (and more general matroids) inspired by a dual notion of chip-firing, and use this to establish Stanley's conjecture for certain classes of graphical matroids.

ART DUVAL, University of Texas at El Paso
A non-partitionable Cohen-Macaulay simplicial complex

In 1979, Richard Stanley posed the following conjecture, which he later described as "a central combinatorial conjecture on Cohen-Macaulay complexes": Conjecture: Every Cohen-Macaulay simplicial complex is partitionable. We disprove this conjecture by constructing an explicit counterexample in three dimensions. Due to a result of Herzog, Jahan and Yassemi, our construction also disproves the conjecture, of great interest in commutative algebra, that the Stanley depth of a monomial ideal is always at least its depth.

This is joint work with Bennet Goeckner, Carly Klivans, and Jeremy Martin.

JENS HARLANDER, Boise State University
On the Homotopy Classification of 2-Complexes

Given a group G and an integer m, how can one find all 2-complexes K (up to homotopy) with fundamental group G and Euler characteristic m? Wesley Browning's work from the late 1970's answered this question in case G is finite (with some exceptions). He showed for example that on the minimal possible Euler characteristic level there can only be finitely many distinct homotopy types, and one can count them. On each level above the minimal one there is a unique homotopy type. Very little is known in case G is infinite. Results obtained by Dunwoody in the early 1980's show that the situation is radically different from the finite case. Even in very concrete settings, for example when G is the Klein bottle group, homotopy classification is a mystery. My talk will be a brief survey of the status of the homotopy classification problem for 2-complexes.

SARA MALONI, University of Virginia
Polyhedra inscribed in quadrics and their geometry.

In 1832 Steiner asked for a characterization of polyhedra which can be inscribed in quadrics. In 1992 Rivin answered in the case of the sphere, using hyperbolic geometry. In this talk, I will describe the complete answer to Steiner's question, which involves the study of interesting analogues of hyperbolic geometry including anti de Sitter geometry. Time permitting, we will also discuss future directions in the study of convex hyperbolic and anti de Sitter manifolds. (This is joint work with J. Danciger and J.-M. Schlenker.)

GABRIEL MINIAN, Universidad de Buenos Aires
Homotopy colimits of finite posets

This is joint work with Ximena Fernandez. I will present a generalization of Thomason's theorem on homotopy colimits over posets, which allows us to characterize homotopy colimits of diagrams of simplicial complexes in terms of the Grothendieck construction on the diagrams of their face posets. We will discuss some analogues of well known results on homotopy colimits in the combinatorial setting, including a cofinality theorem and a generalization of Quillen's Theorem A for posets.

LUIS MONTEJANO, National University of mexico at Queretaro
Homological Sperner-type Theorems

Let K be a simplicial complex. Suppose the vertices of K are painted with $I = \{1, ..., m\}$ colours. A homological Sperner-type theorem concludes the existence of a rainbow simplex of K under the hypothesis that certain homology groups of certain subcomplexes of K are zero. The problem of the existence of a rainbow simplex of K is equivalent to the problem of the existence of a system of distinct representatives in a family of sets.

EDGARDO ROLDÁN-PENSADO, Centro de Ciencias Matemáticas, UNAM
Measure partitions with fixed directions

We prove that, just like polynomials of degree at most $t-1$, paths along two directions (vertical and horizontal) with at most $t-1$ turns can simultaneously split any t measures. This is done by analysing the topology of a specific space of possible partitions. Then we look at other problems, like a generalisation of the the necklace theorem, that may be approached with similar ideas.

STEPHAN ROSEBROCK, Pädagogische Hochschule Karlsruhe, Institut für Mathematk
Labelled Oriented Trees and the Whitehead Conjecture

The Whitehead conjecture asks whether a subcomplex of an aspherical 2-complex is always aspherical. This question is open since 1941. Howie has shown that the existence of a finite counterexample implies (up to the Andrews-Curtis conjecture) the existence of a counterexample within the class of labelled oriented trees. Labelled oriented trees are algebraic generalisations of Wirtinger presentations of knot groups. In this talk we present several possibilities to show asphericity in the class of labelled oriented trees. There are many known classes of aspherical LOTs given by the weight test of Gersten, the I-test of Barmak/Minian, LOTs of Diameter 3 (Howie), LOTs of complexity two (Rosebrock) and several more.

ALEX SUCIU, Northeastern University
Polyhedral products, duality properties, and Cohen-Macaulay complexes

The polyhedral product is a functorial construction that assigns to each simplicial complex K on n vertices, and to each pair of topological spaces, (X, A), a certain subspace, $\mathcal{Z}_K(X, A)$, of the cartesian product of n copies of X. I will discuss some of the relationships between the duality properties of these spaces and the Cohen-Macaulay property of the original simplicial complex. This is based on joint work with Graham Denham and Sergey Yuzvinsky.