Free Probability and its Applications
Probabilité libre et ses applications
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VICTOR PEREZ ABREU, Center for Research in Mathematics CIMAT, Guanajuato, Mexico
On new noncommutative processes arising from matricial random processes

The Dyson-Brownian motion is the process of eigenvalues of a matrix Brownian motion. The limiting of its empirical spectral process, when dimension increases, is the free Brownian motion. In this talk we present new noncommutative processes arising in a similar way from fractional Hermitian Brownian, and fractional Wishart processes.

SOLESNE BOURGUIN, Boston University
Some recent results on Wigner integrals

In this talk, we present recent results dealing with Wigner integrals, which are the building blocks of free Brownian motion functionals. We will discuss limit theorems and quantitative limit theorems involving Wigner integrals, as well as several characterizations of freeness for free random variables having the form of Wigner integrals. If time permits, we will discuss applications of these characterizations to limit theorems.

ISAAC PÉREZ CASTILLO, Institute of Physics, UNAM and London Mathematical Laboratory, UK
Large deviation function for the number of eigenvalues of sparse random graphs inside an interval

We present a general method to obtain the exact rate function $\Psi_{[a,b]}(k)$ controlling the large deviation probability Prob$[\mathcal{I}_N[a,b] = kN] \asymp e^{-N\Psi_{[a,b]}(k)}$ that an $N \times N$ sparse random matrix has $\mathcal{I}_N[a,b] = kN$ eigenvalues inside the interval $[a,b]$. The method is applied to study the eigenvalue statistics in two distinct examples: (i) the shifted index number of eigenvalues for an ensemble of Erdös-Rényi graphs and (ii) the number of eigenvalues within a bounded region of the spectrum for the Anderson model on regular random graphs. A salient feature of the rate function in both cases is that, unlike rotationally invariant random matrices, it is asymmetric with respect to its minimum. The asymmetric character depends on the disorder in a way that is compatible with the distinct eigenvalue statistics corresponding to localized and delocalized eigenstates. The results also show that the level compressibility κ_2/κ_1 for the Anderson model on a regular graph fulfills $0 < \kappa_2/\kappa_1 < 1$ in the bulk regime, in contrast to the behavior found in Gaussian random matrices.

MARIO DIAZ, Queen's University
A New Application of Free Probability Theory: Data Privacy

Many applications of free probability theory have emerged from random matrix theory problems. In this talk we will discuss a new problem connected to the area of data privacy. In addition to our recent developments, we will discuss a conjecture concerning the free multiplicative convolution of a Marchenko-Pastur and a Bernoulli distributions.

– This Is joint work with S. Asoodeh, J. Mingo, S. Belinschi, F. Alajaji, and T. Linder.

YINZHENG GU, Queen's University
Bi-monotonic independence for pairs of algebras

According to Muraki's classification work, there are only five notions of independences in a natural sense: tensor, free, Boolean, monotonic, and anti-monotonic. Following Voiculescu's extension from free to bi-free indpendnece, the notion of Boolean independence has been recently upgraded to bi-Boolean independence as well. In this talk, we consider a similar generalization in the framework of monotonic probability and introduce the notion of bi-monotonic independence for pairs of algebras. Time permitting, we will discuss related topics such as bi-monotonic cumulants, convolution, and a connection with operator-valued monotonic independence.

TODD KEMP, UC San Diego
Partitioned Matrices with Correlations

There is a vast literature on "band matrices" which are symmetric random matrices with independent but not necessarily identically-distributed entries. Recently, I have been studying models where independence is also abandoned.

I will present two kinds of results. First, in joint work with D. Zimmermann, we show that if the matrix entries are partitioned into independent blocks, and if the blocks are not too big, then the empirical spectral distribution still concentrates on its mean as the dimension grows. Second, I will discuss recent and ongoing work with undergraduate research students on several partitioned matrix models whose blocks grow with dimension, where the empirical spectral distribution has a limit which can be computed, using the tools of operator-valued free probability, and other combinatorial and analytic means.

ALEXEY KUZNETSOV, York University
Free stable distributions

I will discuss some analytical properties of free stable distributions, derived using Mellin transform technique. The results include an explicit formula for the Mellin transform, an explicit series representations for the characteristic function and for the density of a free stable distribution. All of these formulas bear close resemblance to the corresponding expressions for classical stable distributions. One consequence of these results is a factorization of a classical stable random variable into an independent (in the classical sense) product of a free stable random variable and a power of a Gamma(2) random variable. This talk is based on joint work with Takahiro Hasebe.

BRENT NELSON, University of California, Berkeley
Free Stein kernels and an improvement of the free logarithmic Sobolev inequality

In their 2015 paper, Ledoux, Nourdin, and Peccati used Stein kernels and Stein discrepancies to improve the classical logarithmic Sobolev inequality (relative to a Gaussian distribution). Simply put, Stein discrepancy measures how far a probability distribution is from the Gaussian distribution by looking at how badly it violates the integration by parts formula. In free probability, free semicircular operators are known to satisfy a corresponding "integration by parts formula" by way of the free difference quotients. Using this fact, we define in this talk the non-commutative analogues of Stein kernels and Stein discrepancies and use them to produce an improvement of Biane and Speicher's free logarithmic Sobolev inequality from 2001. This talk is based on joint work with Max Fathi.

ALEXANDRU NICA, University of Waterloo
An application of free cumulants to meandric systems

I will discuss a class of meandric systems which contains a good number of interesting examples, and where on the other hand the expected number of components can be calculated via arguments concerning free cumulants. This is joint work with Ian Goulden and Doron Puder.

JONATHAN NOVAK, University of California San Diego
Semiclassical asymptotics of $\mathrm{GL}_N(\mathbb{C})$ tensor products

It has been known since seminal work of Biane that the asymptotic behaviour of irreducible representations of the complex general linear group in coupled semiclassical/large-dimension limits is governed by additive free convolution. Biane's original work on this connection required superlinear decay of the semiclassical parameter as a function of N. More recently, Bufetov and Gorin conjectured that linear decay is sufficient. I will present recent work with Collins and Sniady in which we prove an unconditional result: semiclassical limits of tensor products are governed by free probability irrespective of the decay rate of the semiclassical parameter.

JAMES PASCOE, Washington University in St. Louis
Applications of model-realization theory to inverse problems in free probability

Classically, Nevanlinna showed that there was bijection between positive finite Borel measures on the reals and analytic self-maps of the upper half plane which satisfy an asymptotic condition via the Cauchy transform. More recently, analogous problems have been considered in free probability by various authors. That is, there should be a correspondence between noncommutative probability and function theory on a noncommutative upper half plane. We will discuss how to re-frame recent developments in Agler model-realization theory developed on the upper half plane to completely understand the inverse problem in the free probabilistic context. This talk represents joint work with Benjamin Passer and Ryan Tully-Doyle.

EMILY REDELMEIER, ISARA Corporation
Cumulants in the finite-matrix and higher-order free cases

I will discuss an extension of the matrix cumulants first defined by Capitaine and Casalis which follows naturally from their interpretation in connection with diagrammatic methods for computing matrix integrals. The asymptotics of these quantities are the higher-order free cumulants. I will focus on the real and quaternionic cases.

PAUL SKOUFRANIS, York University
Conditional Bi-Free Independence

In this talk, we discuss the recent extension of the notion of bi-free independence to two-state systems. This so called conditional bi-free independence enables one to keep track of more information pertaining to the actions of the left and right regular representations on reduced free product spaces thereby permitting a greater number of non-commutative probabilities to be modelled. The focus of this talk will be the definition of conditional bi-free independence, the combinatorial formula for both the moment and cumulant functions, the operator-valued setting, the partial R-transforms, and infinitely divisible conditional bi-free distributions. (Joint work with Y. Gu.)

PIERRE TARRAGO, Centro de Investigación en Matematicas (Mexico)
Free wreath product quantum groups and free probability

A free wreath product is an algebraic construction which builds a new quantum group from a compact matrix quantum group and a non-commutative permutation group, in the same spirit as the usual wreath product. In this talk, I will present some recent results on the representation theory of certain free wreath products: I will first introduce the notion of vectorial Boolean cumulants for a compact quantum group, and then I will give an explicit basis of the intertwiner spaces of a free wreath product in terms of those vectorial Boolean cumulants.

We significantly use a connection between representation theory of non-commutative permutation groups and planar algebras, and some of our results generalize to arbitrary free products of planar algebras. This is a joint work with Jonas Wahl.

JIUN-CHAU WANG, University of Saskatchewan
Multiplicative bi-free infinite divisibility

We will present a bi-free analogue of Khintchine's characterization of infinite divisibility for commuting left and right unitaries.