Finite and Infinite Dimensional Hamiltonian Systems
Systèmes hamiltoniens en dimension finie et infinie
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RENATO CALLEJA, IIMAS-UNAM
Symmetries and choreographies in families that bifurcate from the polygonal relative equilibrium of the n-body problem

In my talk I will describe numerical continuation and bifurcation techniques in a boundary value setting used to follow Lyapunov families of periodic orbits. These arise from the polygonal system of n bodies in a rotating frame of reference. When the frequency of a Lyapunov orbit and the frequency of the rotating frame have a rational relationship, the orbit is also periodic in the inertial frame. We prove that a dense set of Lyapunov orbits, with frequencies satisfying a Diophantine equation, correspond to choreographies. We present a sample of the many choreographies that we have determined numerically along the Lyapunov families and bifurcating families, namely for the cases n=4,6,7,8 and, 9. We also present numerical results for the case where there is a central body that affects the choreography, but that does not participate in it. This is joint work with Eusebius Doedel and Carlos García Azpeitia.

ANDRES CONTRERAS, NMSU
Eigenvalue preservation for the Beris-Edwards system

Eigenvalue preservation in the Beris-Edwards system is the property that ensures that Q-tensors remain physical along the flow. The Beris-Edwards system is a simplified model for the evolution of nematic liquid crystals. The preservation of eigenvalues property is known to hold for the evolution in the whole space thanks to the work of Xiang-Zarnescu. In this talk I present a simpler and shorter proof that also applies to the bounded domain case. This is joint work with Xiang Xu and Wujun Zhang.

LIVIA CORSI, Georgia Institute of Technology
A non-separable locally integrable Hamiltonian system

A mechanical Hamiltonian on $\mathbf{T}^2 \times \mathbf{R}^2$ is said to be "separable" if in some coordinate system (q,p) it has the form

$$H(q,p) = \frac{1}{2}|p|^2 + V_1(q_1) + V_2(q_2).$$

Clearly such an Hamiltonian is globally Liouville-integrable. I will show that there exists an analytic, non-separable, mechanical Hamiltonian which is only locally integrable. Precisely I will show that H is integrable on an open subset $\mathcal{U}$ of the energy surface $\mathcal{S} := \{H = 1/2\}$, whereas on $\mathcal{S} \setminus \mathcal{U}$ it exhibits cahotic behavior. This is a joint work with V. Kaloshin

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WALTER CRAIG, McMaster University
A Hamiltonian and its Birkhoff normal form for water waves

A 1968 paper by VE Zakharov gives a formulation of the equations for water waves as a Hamiltonian dynamical system, and shows that the equilibrium solution is an elliptic stationary point. This talk will discuss two aspects of the water wave equations in this context. Firstly, we generalize the formulation of Zakharov to include overturning wave profiles, answering a question posed to the speaker by T. Nishida. Secondly, we will discuss the question of Birkhoff normal forms for the water waves equations in the setting of spatially periodic solutions. We transform the water wave problem with nonzero surface tension to third order Birkhoff normal form, and in the case of zero surface tension in deep water, to fourth order Birkhoff normal form. The result includes a discussion of the dynamics of the normal form, and a quantification of the function space mapping properties of these transformations.

NICHOLAS FAULKNER, UOIT (University of Ontario)
Equivariant KAM

Kolmogorov-Arnold-Moser (KAM) theory has a rich and well developed history. In this talk we present a KAM theory for Γ-equivariant Hamiltonian systems. Hamiltonian systems with discrete symmetry groups Γ arise naturally in many settings including for instances the N-body problem. If Γ is Abelian, then KAM theorem applies, but for Γ non-Abelian, $1:1$ resonance effects lead to small divisor problems. These problems can be overcome by combining the isotypic decomposition of phase space with a detailed study of Γ and Torus invariants, all within the classical iterative proof structure. This is joint work with Dr. Luciano Buono.

SLIM IBRAHIM, University of Victoria
Ground State Solutions of the Gross Pitaevskii Equation Associated to Exciton-Polariton Bose-Einstein Condensates

We investigate the existence of ground state solutions of a Gross-Pitaevskii equation modeling the dynamics of pumped Bose Einstein condensates (BEC). The main interest in such BEC comes from its important nature as macroscopic quantum system, constituting an excellent alternative to the classical condensates which are hard to realize because of the very low temperature required. Nevertheless, the Gross Pitaevskii equation governing the new condensates presents some mathematical challenges due to the presence of the pumping and damping terms. Following a self-contained approach, we prove the existence of ground state solutions of this equation under suitable assumptions: This is equivalent to say that condensation occurs in these situations. We also solve the Cauchy problem of the Nonlinear Schrödinger equation and prove some corresponding laws

ROBERT L. JERRARD, University of Toronto
dynamics of nearly-parallel vortex filaments in the Gross-Pitaevskii equations

We study the motion of thin, nearly parallel vortex filaments in 3d solutions of the Gross-Pitaevskii equations. In particular, we show that in a certain scaling limit, these filaments are governed by a system of nonlinear Schrödinger equations formally derived by Klein, Majda, and Damodaran in the mid '90s in the context of the Euler equations. This is the first rigorous justification of the Klein-Majda-Damodaran model in any setting. This is joint work with Didier Smets.

STEFAN LE COZ, University of Toulouse
Stability of multi-solitons for the derivative nonlinear Schrödinger equation

The nonlinear Schrödinger equation with derivative cubic nonlinearity (dNLS) is a model quasilinear dispersive equation. It admits a family of solitons, which are orbitally stable in the energy space. After a review of the many interesting properties of dNLS, we will present a result of orbital stability of multi-solitons configurations in the energy space, and some ingredients of the proof.

ROSA MARÍA VARGAS MAGAÑA, UNIVERSIDAD NACIONAL AUTONÓMA DE MÉXICO
Whitham-Boussinesq model for variable depth topography. Results on normal and trapped modes for non trivial geometries.

The water-wave problem describes the evolution of an incompressible ideal, irrotational fluid with a free surface under the influence of gravity. A significant development in water-wave theory was the discovery by Zakharov in 1968 that the problem has a Hamiltonian structure and later W. Craig and C. Sulem introduced the Dirichlet-Neumann operator explicitly on the Hamiltonian. In this talk I will present a joint work with Prof. Panayotis Panayotaros and Prof. Antonmaria Minzoni from Universidad Nacional Autónoma de México, we propose a simplified long wave model combining a variable depth generalization of the exact nonlocal dispersion with the standard Boussinesq nonlinearity. The model relies on an approximate Dirichlet-Neumann operator that preserves some key structural properties of the exact operator and is simpler than alternative perturbative or implicit expressions. We examine the accuracy of this approximation by studying linear (2-D) normal modes and (3-D) longitudinal and Ursell modes for some geometries for which there are exact results.

MICHELA PROCESI, Universita' di Roma Tre
Finite dimensional invariant tori in PDEs

I shall discuss the existence and stability of quasi-periodic invariant tori for classes of evolution PDEs, both in Hamiltonian and reversible setting,trying to give an idea of the general strategy and the main difficulties in problems of this kind. I will also discuss the related problem of unstable solutions.

GEORDIE RICHARDS, Utah State University
On invariant Gibbs measures for generalized KdV

We will discuss some recent results on invariant Gibbs measures for the periodic generalized KdV equations (gKdV). Proving invariance of the Gibbs measure for gKdV is nontrivial due to the low regularity of functions in the support of this measure. Bourgain proved this invariance for KdV and mKdV, which have quadratic and cubic nonlinearities, respectively. Previously, we proved invariance of the Gibbs measure for the quartic gKdV by exploiting a nonlinear smoothing induced by initial data randomization. More recently, in joint work with Tadahiro Oh (Edinburgh) and Laurent Thomann (Lorraine), we have established this invariance for gKdV with any odd power (defocusing) nonlinearity. The proof relies on a probabilistic construction of solutions using the Skorokhod representation theorem.

YANNICK SIRE, Johns Hopkins University
A posteriori KAM for PDEs

I will describe recent results in collaboration with R. de la Llave on an a posteriori KAM method for PDEs. In particular, our methods uses very little of symplectic geometry and does not use transformation theory. It applies to ill-posed equations in the Hadamard sense and we will give applications to the so-called Boussinesq equation by constructing periodic solutions for it.

LUIS VEGA, Universidad del País Vasco

CHENG YU, University of Texas at Austin