Fractal Geometry and Dynamical Systems
Géométrie fractale et systèmes dynamiques
(Org: **Carlos Gustavo Moreira** (IMPA), **Pablo Shmerkin** (University of Torcuato), **Károly Simon** (University of Technology and Economics) and/et **Boris Solomyak** (Bar-Ilan University))

BALÁSZ BÁRÁNY, Einstein Inst. Mathematics, Hebrew Univ. of Jerusalem & MTA-BME Stochastic Res. Group, BUTE
Dimension of self-affine sets with typical linear parts

An affine iterated function system is a finite collection of affine invertible contractions and the invariant set associated to the mappings is called self-affine. In 1988, Falconer proved that, for given matrices, the Hausdorff dimension of the self-affine set is the affinity dimension for Lebesgue almost every translation vectors. Similar statement was proven by Jordan, Pollicott, and Simon in 2007 for the dimension of self-affine measures. They also introduced a self-affine transversality condition, which allowed to consider parametrised translation vectors.

In this talk, we have an orthogonal approach. We introduce a modified self-affine transversality condition, which allows us to consider a family of parametrised linear parts. Moreover, we show a class of self-affine systems in which, for given translation vectors, we get the same results, like Falconer'88 and Jordan, Pollicott and Simon'07, for Lebesgue almost all matrices.

This is a joint work with Antti Käenmäki and Henna Koivusalo.

ANTON GORODETSKI, University of California Irvine
Separable potentials and sums of regular Cantor sets.

Questions on structure of sums of Cantor sets appear naturally in spectral theory. Namely, spectra of discrete Schrodinger operators on two dimensional lattice is the sum of spectra of corresponding operators on one dimensional lattice, while for a large class of operators in one dimensional case the spectrum is known to be a Cantor set. We will discuss these relations, and the series of recent results (joint with D.Damanik and B.Solomyak) on the subject.

SERGIO AUGUSTO ROMAÑA IBARRA, Universidade Federal do Rio de Janeiro
On the Lagrange and Markov Dynamical Spectrum for Surfaces of Negative Curvature

Let X be a complete vector field on the a surface M.
Given a continuous real function $f\colon M \to \mathbb{R}$, we define the Lagrange dynamical spectra associated to (f, X) by

$$L(f,X) = \left\{ \limsup_{t\to\infty} f(X^t(x)) : x \in M \right\},$$

and the Markov Dynamical spectra associated to (f, X) by

$$M(f,X) = \left\{ \sup_{t\in\mathbb{R}} f(X^t(x)) : x \in M \right\}.$$

In this lecture we show that for a typical non-compact surface of finite volume and negative curvature the Lagrange and Markov dynamical spectra associated to the geodesic flow have non-empty interior for a "large" set of the real functions on the the surface.

MEHDI POURBARAT, Department of Mathematics, Shahid Beheshti University, Tehran, Iran
On the arithmetic difference of middle Cantor sets

We discuss about the arithmetic difference of affine Cantor sets defined by the simplest possible combinatorics. We determine all triples (α, β, λ) such that $C_\alpha - \lambda C_\beta$ forms a closed interval, where C_α and C_β are middle Cantor sets. In the case $\lambda = -1$, we extend a result of Mendes and Oliveira. Among the other results, prototype examples of middle Cantor sets C_α and C_β are introduced such that the sets $C_\alpha \cdot C_\beta$ and C_α / C_β contain an interval, while the product of their thickness is smaller than one.

In sequel, a new family of triples $(C_\alpha, C_\beta, \lambda)$ is indicated for which $C_\alpha - \lambda C_\beta$ contains an interval or has zero Lebesgue measure. A special triple $(C_\alpha, C_\beta, \lambda)$ is selected and the iterated function system corresponding to the attractor $C_\alpha - \lambda C_\beta$ has been characterized. Some specifications of the attractor has been presented that keeps our example as an exception among others.

CARLOS MATHEUS SILVA SANTOS, CNRS, Universite Paris 13

On the Lagrange and Markov spectrum

The complement $M \setminus L$ of the Lagrange spectrum L in the Markov spectrum M was studied by many authors (including Freiman, Berstein, Cusick and Flahive). After their works, we dispose of a countable collection of points in $M \setminus L$.

In this talk, we describe the structure of $M \setminus L$ near a non-isolated point α_∞ found by Freiman in 1973, and we use this description to exhibit a concrete Cantor set whose Hausdorff dimension coincides with the Hausdorff dimension of $M\setminus$ near α_∞. A consequence of our results is the lower bound $HD(M \setminus L) > 0.353$ on the Hausdorff dimension of $M \setminus L$. This is a joint work with C. G. Moreira.

PABLO SHMERKIN, Torcuato Di Tella University and CONICET

Furstenberg's intersection conjecture and the L^q norm of convolutions

I will describe some of the steps involved in the recent solution of Furstenberg's 1969 conjecture on the dimensions of the intersections of sets invariant under multiplication by 2 and by 3 on the circle (a completely different solution was independently obtained by Meng Wu). In particular, I will focus on an inverse theorem for the flattening of the L^q norm of discrete measures under convolution, which is one of the key tools and may have other applications.

KRYSTAL TAYLOR, The Ohio State

On the algebraic sum of a planar set and a smooth curve

Given a set $A \subset \mathbb{R}^2$, we study the set of those points on the plane which are at a distance 1 from at least one of the elements of A, where "distance" means either the Euclidean distance or some other natural distances on the plane. This set is $A + S^1$, where S^1 is the unit circle in the given distance. More generally, we consider $A + \Gamma$, for a suitable curve Γ. We provide a variety of conditions which guarantee that $A + \Gamma$ is big in the sense that it contains interior points. A connected problem is to the study of pinned distance sets. We also prove that the pinned distance set of $C \times C$, where C is a sufficiently thick Cantor set has interior. This is joint work with Karoly Simon.