Equations of Fluid Mechanics: Numerics
Équations de la mécanique des fluides : analyse numérique

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ALEXANDER BIHLO, Memorial University of Newfoundland

A well-balanced meshless tsunami propagation and inundation model

We derive a universal criterion for the preservation of the lake at rest solution in general mesh-based and meshless numerical schemes for the shallow-water equations with bottom topography. The main idea is a careful mimetic design for the spatial derivative operators in the momentum flux equation that is paired with a compatible averaging rule for the water column height arising in the bottom topography source term. The resulting numerical schemes for the shallow-water equations are called well-balanced.

Based on a well-balanced RBF-FD discretization of the shallow-water equations, we develop a meshless tsunami propagation and inundation model. The moving wet-dry interface between the incoming wave and the shoreline is handled using RBF generated extrapolation, yielding a truly meshless tsunami model. Several numerical results are presented that demonstrate excellent agreement of the resulting model with standard one- and two-dimensional benchmark tests.

This is joint work with Rüdiger Brecht, Scott MacLachlan and Jörn Behrens.

JOHN BOWMAN, University of Alberta

On the Global Attractor of 2D Incompressible Turbulence with Random Forcing

We revisit bounds on the projection of the global attractor in the energy–enstrophy plane obtained by Dascaliuc, Foias, and Jolly [2005, 2010]. In addition to providing more elegant proofs of some of the required nonlinear identities, the treatment is extended from the case of constant forcing to the more realistic case of random forcing. Numerical simulations in particular often use a stochastic white-noise forcing to achieve a prescribed mean energy injection rate. The analytical bounds are illustrated numerically for the case of white-noise forcing.

ALEXANDRE NOLL MARQUES, MIT

Solving flow problems to high order of accuracy with embedded boundaries

In this talk I present a new method to solve flow problems with computational grids that are not aligned with boundaries and interfaces. In many applications, flexible boundaries, or interfaces between different phases, interact dynamically with the underlying flow and change over time. These applications pose many challenges to numerical methods that rely on body-fitted grids, sparking the interest in methods where boundaries and interfaces are embedded into simple computational grids. The key features that distinguish the new method from other embedded boundary methods are (i) high order of accuracy, (ii) compact discretization stencils, (iii) cost-effectiveness, and (iv) robustness. I discuss the details of a fourth implementation of the method for elliptic equations with discontinuity interfaces, and show applications to incompressible flows.

GEOFF MCGREGOR, McGill University

A Parametric Interpolation Framework for 1D Scalar Conservation Laws using the Equal Area Principle

The equal area principle is a technique used for locating shocks in weak solutions of scalar conservation laws. Despite widespread knowledge of the equal area principle, little work has been done to make it viable from a numerical standpoint. In this talk we present key analytical results which allows weak solutions to be obtained purely from solutions of a parametric interpolation problem. We will discuss properties of our proposed parametric interpolation framework, along with numerical results and plans for future work.

JEAN-CHRISTOPHE NAVE, McGill University
Solving Incompressible 2D Euler's Equations with Exponential Resolution

In this talk I will present a new method to solve the incompressible 2D Euler's equations. The proposed approach exploits the geometrical structure of the equations. The end result is an algorithm which runs in linear-time and possesses exponential resolution, thus able to represent arbitrary small-size features in the solution.

PASCAL POULLET, Université des Antilles, Guadeloupe FWI
An explicit predictor-corrector scheme for sediment transports

In this talk, I will introduce a residual distribution scheme for computing bedload sediment transports in a shallow water environment. The approach that has been chosen, is to solve a hyperbolic nonlinear partial differential system of equations in a nonconservative form. Moreover, as we aim to obtain an efficient procedure including a shock capturing strategy, we consider an explicit predictor-corrector scheme with a flux limiter. During the presentation, particular attention will be made on accuracy, stability, well-balanced property and the implementation on clusters.

LEO REBHOLZ, Clemson University
On conservation laws of Navier-Stokes Galerkin discretizations

We study conservation properties of Galerkin methods for the incompressible Navier-Stokes equations, without the divergence constraint strongly enforced. In typical discretizations such as the mixed finite element method, the conservation of mass is enforced only weakly, and this leads to discrete solutions which may not conserve energy, momentum, angular momentum, helicity, or vorticity, even though the physics of the Navier-Stokes equations dictate that they should. We aim in this work to construct discrete formulations that conserve as many physical laws as possible without utilizing a strong enforcement of the divergence constraint, and doing so leads us to a new formulation that conserves each of energy, momentum, angular momentum, enstrophy in 2D, helicity and vorticity (for reference, the usual convective formulation does not conserve most of these quantities). Several numerical experiments are performed, which verify the theory and test the new formulation.

DAVID SHIROKOFF, New Jersey Institute of Technology
Unconditional Stability for Multistep Imex Schemes

In this talk we introduce a new class of linear multistep ImEx schemes that have very good unconditional stability properties. Unconditional stability is a desirable property of a time stepping scheme, as it allows the choice of time step solely based on accuracy considerations. Of particular interest are problems for which both the implicit and explicit parts of the ImEx splitting are stiff. Such splittings can arise, for example, in variable-coefficient problems, or the incompressible Navier-Stokes equations. To characterize the new ImEx schemes, we introduce an unconditional stability region, which plays a role analogous to that of the stability region in conventional multistep methods. We show how this region may be characterized through the use of a conformal mapping. Moreover, we will show how the new diagrams explain the fundamental stability restrictions of the well-known semi-implicit backward differentiation formulas (SBDF). We further show that the new ImEx coefficients can overcome the limitations of SBDF, and highlight their utility with several examples arising from partial differential equations: such as variable diffusion, advection diffusion and, time permitting a time dependent Stokes equation.

ANDY WAN, McGill University
Conservative schemes for dynamical systems with application to vortex dynamics

We present a new class of conservative method, called the multiplier method, which enables systematic construction of conservative schemes for general dynamical systems. Specifically, the multiplier method can preserve arbitrary forms of conserved quantities and is applicable for systems without a symplectic or variational structure, such as dissipative problems. We illustrate this method for the point vortex problem on the plane and the sphere, and if time permits, for vortex blobs dynamics.