
Current Trends in Combinatorics
Tendances actuelles en combinatoire

(Org: **Alan Arroyo** (University of Waterloo), **Adriana Hansberg** (Instituto de Matemáticas, UNAM Juriquilla), **Bruce Richter** (University of Waterloo) and/et **Gelasio Salazar** (Universidad Autónoma de San Luis Potosí))

SILVIA FERNÁNDEZ, California State University, Northridge
Rectilinear Local Crossing Numbers of Complete and Complete Bipartite Graphs

The local crossing number of a drawing of a graph is the largest number of crossings on any edge of the drawing. In a rectilinear drawing of a graph, the vertices are points in the plane in general position and the edges are straight line segments. The rectilinear local crossing number of a graph G is the minimum local crossing number over all rectilinear drawings of G . In this talk, we present the latest results on rectilinear local crossing numbers of the complete and the complete bipartite graphs.

JACOB FOX, Stanford University
Regularity methods in discrete geometry

In this talk, I will discuss variants of Szemerédi's celebrated graph regularity lemma with much better quantitative bounds, and how they can be used to make progress on problems in discrete and computational geometry. Based on joint work with János Pach and Andrew Suk.

PENNY HAXELL, University of Waterloo
Stability for matchings in regular tripartite hypergraphs

We consider a hypergraph analogue of the basic fact that every regular bipartite graph has a perfect matching. A theorem of Aharoni implies that every regular tripartite hypergraph H with n vertices in each class has a matching of size at least $n/2$, and moreover this bound is tight. We prove a stability version of this statement, showing that if H has matching number close to $n/2$ then it is correspondingly close in structure to the extremal configuration.

MARCOS KIWI, U. Chile
Bijjective Proof of Kasteleyn's Toroidal Perfect Matching Cancellation Theorem

Kasteleyn (1961) showed how to express the dimer partition function for the square lattice on the torus using 4 Pfaffians and stated without proof that the partition function for a graph of genus g requires 2^{2g} Pfaffians (a fact rigorously established almost four decades later).

Via algebraic considerations, Kasteleyn also showed that for the square lattice, under some mild conditions on the edge weights, one of the 4 Pfaffians vanishes (equals 0). It is not evident if and how a similar situation occurs for square lattices of genus $g > 1$. We give a combinatorial proof of Kasteleyn's observation for $g = 1$. We also discuss potential extensions of our result for the higher order genus case and briefly elaborate on possible implications concerning the widely believed assumption in theoretical physics that dimer models are discrete analogues of free fermions.

Based on joint work with A. Jiménez (U. Valparaíso, Chile) and M. Loebl (Charles U., Czech Republic)

SULAMITA KLEIN, Universidade Federal do Rio de Janeiro
Graph Sandwich Problems and Graphs Partition Problems: how do they relate?

In this talk we consider two decision problems. The first one is the Golumbic, Kaplan and Shamir decision sandwich problem for the property II, where two graphs $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ are given, such that $E^1 \subseteq E^2$, plus the question whether there exists a graph $G = (V, E)$, such that $E^1 \subseteq E \subseteq E^2$, and G satisfies property II. The second one is the Feder, Hell,

Klein and Motwani decision graph partition problem, where it is given a graph G and the question whether there is a partition for the vertex set V of G into at most k sets V_1, V_2, \dots, V_k , which can have inner properties (like V_i being a clique, or an independent set, or no restriction) and external properties (like $V_i V_j$ being completely adjacent, or completely non-adjacent, or no restriction).

We will discuss the complexity of the sandwich problem for properties or classes of graphs arising from partition problems.

ALEXANDR KOSTOCHKA, University of Illinois at Urbana-Champaign

Packing chromatic number of subcubic graphs

A packing k -coloring of a graph G is a partition of $V(G)$ into sets V_1, \dots, V_k such that for each $1 \leq i \leq k$, the distance between any two distinct $x, y \in V_i$ is at least $i + 1$. The packing chromatic number, $\chi_p(G)$, of a graph G is the minimum k such that G has a packing k -coloring. Sloper showed that there are 4-regular graphs with arbitrarily large packing chromatic number. The question whether the packing chromatic number of subcubic graphs is bounded appears in several papers. We answer this question in the negative. Moreover, we show that for every fixed k and $g \geq 2k + 2$, almost every n -vertex cubic graph of girth at least g has the packing chromatic number greater than k . This is joint work with J. Balogh and X. Liu.

AMANDA MONTEJANO, Science Faculty UNAM

The existence of zero-sum subgraphs in $\{-1, 1\}$ -weightings of $E(K_n)$

For a given graph H , and a $\{-1, 1\}$ -weighting function f of the edges of the complete graph K_n , we study the existence of a zero-sum copy of H ; that is, a copy of H in K_n with $\sum_{e \in E(H)} f(e) = 0$. For example, for all $n \geq 5$, we can guarantee the existence of a zero-sum copy of K_4 for every $f : E(K_n) \rightarrow \{-1, 1\}$ if we are provided with enough edges of both types; however, this phenomenon is not true if H is a complete graph on $m \geq 5$ vertices. So, given a graph H , our aim is to determine a function $h(H, n)$, if it exists, such that, for sufficiently large n , every $f : E(K_n) \rightarrow \{-1, 1\}$ with $\min\{e(-1), e(1)\} \geq h(H, n)$ contains a zero-sum copy of H , where $e(-1)$ and $e(1)$ denote the number of edges assigned -1 and 1 , respectively. In the cases where H is a path, a star or a tree, the exact function $h(H, n)$ is determined and the extremal $\{-1, 1\}$ -weighting functions are characterized.

Of course, such type of problems can be regarded as edge coloring problems: while Ramsey Theory studies the existence of monochromatic subgraphs in edge colorings on the complete graph, and anti-Ramsey Theory studies the existence of rainbow subgraphs, here we study the existence of *balanced* subgraphs, where balanced means to have the same number of edges of each color.

This is a joint work with Yair Caro and Adriana Hansberg.

JUAN JOSÉ MONTELLANO, Instituto de Matemáticas, UNAM

Transversals of sequences

Given an n -set M and a positive integer $k \leq n$, as a k -sequence of elements of M we will understand an injective function $G : \{1, \dots, k\} \rightarrow M$. A k -sequence G will be also denoted as (x_1, \dots, x_k) , with $G(i) = x_i$ for each $i = 1 \dots, k$. A set T of k -sequences of M will be called an (n, k) -transversal if for each n -sequence $S = (x_1, \dots, x_n)$ of elements of M , there is $(y_1, \dots, y_k) \in T$ such that (y_1, \dots, y_k) is a subsequence of S . An instance of this notion of transversal can be deduce from a result of Erdős and Szekeres which states that every sequence of n integers contains a crescent or decrescent subsequence of order k (for $k \leq \lceil \sqrt{n} \rceil$). Given an n -set M and a digraph $D = (V, A)$, with vertex-set $M = V$, the set of transitive tournaments of order k of D define, in a "natural way", a set of k -sequences of M (each of those transitive tournaments of D can define two sequences of k elements of M). In this talk we present some (n, k) -transversals which arise from the set of transitive tournaments of digraphs, some of them closely related with the diagonal Ramsey numbers.

ROB MORRIS, IMPA

The typical structure of sparse graphs in hereditary properties

The method of hypergraph containers, introduced a few years ago by Balogh, Morris and Samotij, and independently by Saxton and Thomason, has proved to be an extremely useful tool in the study of the typical properties of sparse H -free graphs. In particular, a fairly straightforward application of this technique allows one to locate the threshold at which the structure of a typical H -free graph transitions from being "random-like" to being "structured". For most hereditary graph properties, however, the standard version of this method does not allow one to establish even the existence of such a threshold. In this talk we will discuss a refinement of the container method that takes into account the asymmetry between edges and non-edges in a sparse member of a hereditary graph property. As an application, we will show how to determine the approximate structure of a typical graph with n vertices, m edges, and no induced copy of C_4 , for all $m \gg n^{4/3}(\log n)^3$.

This is based on joint work with Wojciech Samotij and David Saxton.

SERGEY NORIN, McGill University

Asymptotic density of graphs excluding a disconnected minor

For a graph H , let

$$c_\infty(H) = \lim_{n \rightarrow \infty} (\max |E(G)|/n),$$

where the maximum is taken over all graphs G on n vertices not containing H as a minor. Thus $c_\infty(H)$ is the asymptotic maximum density of graphs not containing an H -minor. Employing a structural result of Eppstein, we prove new upper bounds on $c_\infty(H)$ for disconnected graphs H .

The case when $H = sK_r$, that is when H is a disjoint union of s copies of the complete graph on r vertices, is of particular interest. Thomason has shown that

$$c_\infty(sK_r) = s(r - 1) - 1$$

for $s = \Omega(r\sqrt{\log r})$. We show that the same conclusion holds for $s = \Omega(\log^{3/2} r)$, which is best possible up to the power of the logarithm.

Based on joint work with Yingjie Qian.

ORTRUD OELLERMANN, University of Winnipeg

On the Mean Order of Certain Types of Substructures of Graphs

The problem of finding the mean order of the subtrees of a tree was introduced by Jamison in 1983. It is known that paths have the smallest mean subtree order among trees of a fixed order. However, the problem of determining the structure of trees with maximum subtree order remains open. We discuss some progress on the latter for subclasses of trees. We also present some extensions of the mean subtree order of a tree to other classes of graphs and present results for these new invariants. (This is joint work with Lucas Mol and Alexander Stephens.)

MARTÍN SAFE, Departamento de Matemática, Universidad Nacional del Sur, Argentina

Characterizing and finding forbidden subgraphs for subclasses of circular-arc graphs

The intersection graph of a set \mathcal{A} of arcs on a circle is a graph having one vertex for each arc in \mathcal{A} and such that two different vertices are adjacent if and only if the corresponding arcs have nonempty intersection. A graph G is a circular-arc graph if and only if G is the intersection graph of some set \mathcal{A} of arcs on a circle; if so, the set \mathcal{A} is called a circular-arc model of G . Forbidden structures for the class of circular-arc graphs and its main subclasses (including subclasses of interval graphs), as well as efficient algorithms for finding these forbidden structures, have received a great deal of attention. In this work, we will present some recent results regarding minimal forbidden induced subgraph characterizations and linear-time algorithms for finding an instance of such forbidden subgraphs for some subclasses of circular-arc graphs, including normal Helly circular-arc graphs, concave-round graphs (also known as Tucker circular-arc graphs or Γ circular-arc graphs), and Helly circular-arc graphs. Results on normal Helly circular-arc graphs are joint work with Yixin Cao and Luciano N. Grippo.

MAYA STEIN,

Partitions into monochromatic subgraphs

The central problem in monochromatic subgraph partitions is the following: given a complete graph on n vertices whose edges are coloured with r colours, the goal is to find a small set of vertex-disjoint monochromatic cycles which together cover all the vertices. There are variants substituting complete graphs with other host graphs, cycles with other classes of graphs, graphs with hypergraphs, and other. The problem goes back to work by Gyarfás from the late 1960's, but the core problem as well as its variants have attracted much interest during the past decade. In this talk, we will give an overview of the most important results and methods of the area, and then discuss some recent results for hypergraphs, which are joint work with Bustamante and Han, and with Bustamante.

YOSHIKO WAKABAYASHI, University of São Paulo, Brazil

Decomposing highly connected graphs into paths of any given length

In 2006, Barát and Thomassen made the following conjecture: for each tree T , there exists a natural number k_T such that, if G is a k_T -edge-connected graph and $|E(T)|$ divides $|E(G)|$, then G can be edge-decomposed into copies of T . In a series of papers, starting in 2008, Thomassen verified this conjecture for stars, some bistars, paths of length 3, and paths whose length is a power of 2. In 2014, we verified this conjecture for paths of length 5, and subsequently, for paths of any given length. In this talk we address this last result. We note that further results on this conjecture have been obtained in the last two years: Bensmail, Harutyunyan, Le and Thomassé (2015) proved this conjecture for paths, using a different approach and weakening the condition on high edge-connectivity; more recently, these authors, together with M. Merker proved the conjecture.

This is joint work with F. Botler, G. O. Mota, and M. T. I. Oshiro, from University of São Paulo, Brazil.