CMS-Studc Student Research Session
Session de recherche étudiante - SMC-comÉtud

OLIVIER BINETTE, Université du Québec à Montréal
Bayesian learning: semiparametric modelling and asymptotic theory

Bayesian learning, for our purposes, is about finding the probability distribution that a stochastic mechanism is following. For instance, it is of interest in bioinformatics to learn the distributions of protein torsional angles to better predict new protein structures. The bayesian solution is to characterize the space of possible distributions and to assign subsets prior probabilities, quantifying uncertainty about what the true distribution may be. Observations are incorporated in the model through Bayes' theorem.

Exploiting a correspondence between a class of bayesian statistical models and linear approximation operators, we demonstrate a general result on the asymptotic correctness of bayesian learning. The practical use of our approach is shown for the problem of learning the distribution of angular data: new angular data models, satisfying smoothness and geometric constraints, are constructed and studied using the approximation-theoric framework.

NITIN CHIDAMBARAM, University of Alberta
Topological recursion and quantum curves

The Eynard-Orantin topological recursion is a technique inspired by random matrix theory, that can be used to compute interesting enumerative invariants including knot invariants, Hurwitz numbers and Gromov-Witten invariants. Given a spectral curve, topological recursion computes an infinite sequence of symplectic-invariant meromorphic differentials on the curve. I will present the basic formalism of topological recursion. A 'quantum curve' is a differential operator associated with the curve and a WKB-type asymptotic solution to this curve. Quantum curves also encode some interesting (quantum) enumerative invariants. In this context, the conjecture that topological recursion on a spectral curve can be used to construct a quantum curve seems natural (this is proved for a large class of genus zero curves). I shall try to explain the relationship between topological recursion and quantum curves and hopefully discuss our recent work with Bouchard and Dauphinee (arxiv:1610.00225) where we studied the first class of higher genus quantum curves.

BRICEYDA DELGADO, Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional
Hilbert Transform associated to the main Vekua equation

We will recall some results from article $[1]$, where an explicit representation to the Hilbert transform $\mathcal{H}$ was given on the unit sphere in $\mathbb{R}^n$. The Hilbert transform takes data on the boundary and completes them to obtain the boundary value of a monogenic (hyper-holomorphic) function. These results will be reduced to the case $n = 3$ with $\Omega = B^3$ and $\partial\Omega = S^2$.

Then, we will give a definition for the Hilbert transform $\mathcal{H}_f$ associated to the main Vekua equation $DW = \frac{Df}{f}\overline{W}$. Furthermore, some results about $\mathcal{H}_f$ will be enunciated such as the continuity, invertibility and uniqueness.

After that an extension in Ω will be given for the vectorial part of the solutions of the main Vekua equation, when the boundary value of the scalar part is known. That vectorial extension in terms of $\mathcal{H}_f$ will be compared with the vectorial extension derivated of the general solution of the Div-Curl system corresponding to the main Vekua equation given in [2].

[1] Tao Qian and Yan Yang. Hilbert Transform on the Sphere with the Clifford Algebra Setting, Journal of Fourier Analysis and Applications, (2009), $753 - 774$.

[2] Briceyda B. Delgado and R. Michael Porter. General Solution of the Inhomogeneous Div-Curl System and Consequences, 2017 (in preparation).

MIRIAM BOCARDO GASPAR, CINVESTAV, México
Local zeta functions and p-adic string amplitudes

In this talk we explain some connections between p-adic local zeta functions and p-adic Koba-Nielsen type string amplitudes. The convergence of these amplitudes is related with the convergence of Igusa-type integrals depending on several complex parameters. Hence, the string amplitudes are "essentially" local zeta functions, and thus, they are algebraic-geometric objects that can be studied over several ground fields, for instance $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}_p$, $\mathbb{C}((t))$, and that on each of these fields these objects have similar mathematical properties.

THAD JANISSE, University of Toronto
Finding Subalgebras of Real Semisimple Lie Algebras

In the 1950's, Tits gave a Bourbaki seminar talk wherein he presented a complete classification of subalgebras of compact real Lie algebras. One of the centrals idea in the classification, regularity, showed that compact Lie algebras have very rigid subalgebra structure. Continuing this line of inquiry, we shall describe techniques to exploit the rigid structure for compact Lie algebras to identify and classify subalgebras in the non-compact case, using the split real form G_2 as an example.

ARPITA KAR, Queen's University
A conjecture of Bateman regarding $r_5(n)$

Paul T. Bateman made a conjecture about the exact formula for $\sum_{|j|\leq n}\sigma(n-j^2)$. Later, he and Knopp proved the conjecture using the theory of half-integral weight modular forms. We present a simpler proof of the conjecture in the case when $n \equiv 1 (\mathrm{mod}\ 4)$. This is joint work with Prof. Ram Murty.

JORDAN KOSTIUK, University of Alberta
Geometrization of Supersymmetry Algebras

Supersymmetry algebras are special kinds of algebras that arise in the study of supersymmetric physics. M.Faux and S.J.Gates have shown that much of the structure of 1-dimensional supersymmetry algebras can be encoded in bipartite graphs with additional combinatorial structures know as *adinkras*. In this talk I will present a method for geometrizing adinkras — that is, a means of interpreting all of the structure of an adinkra in a geometrically meaningful way — as described in my recent works. After describing the construction I will spend some time explaining just how special the kinds of objects that we are deriving in this way are; this is a reflection of the supersymmetry from which we started.

MARIE LAFRANCE, Université de Montréal
Supersymmetric sigma models and constant solutions

It was recently shown how to construct surfaces immersed in the $su(N)$ Lie algebra from solutions of the bosonic CP^{N-1} sigma model. The method has also been generalized to more general Grassmanniann sigma models $G(M,N)$. Furthermore, we know in many cases the conditions on the solutions to lead to constant gaussian curvature surfaces. In this presentation, I will show you how to generalize these conditions to supersymmetric fields. The work has been done for the case CP^{N-1} but little is known for other Grassmanniann manifolds fields. Starting with Lagrange equations, I will construct the extended solutions and prove that they have constant curvature. In some cases, we can prove that these are the only constant curvature solutions that exists.

JEAN LAGACÉ, Université de Montréal
Lattice point counting in spectral theory

The spectral asymptotics of elliptic operators (e.g. the Laplacian) and analytic number theory are linked together via lattice point counting problems. As an example, counting eigenvalues of the Laplacian on a flat square torus corresponds to counting integer points in a disk of large radius, which is precisely the Gauss circle problem, a longstanding problem in analytic number theory.

In this presentation, I will explain how lattice counting methods are applied to study spectral asymptotics for Schrödinger operators on waveguides and resonators, and also for the Steklov eigenvalue problem on a cube. These two settings will illustrate different flavors of lattice counting problems arising in spectral theory. The talk is based on joint works with L. Parnovski (UCL), as well as with A. Girouard (Laval), I. Polterovich (Montréal) and A. Savo (Rome).

SUMIN LEEM, University of Calgary
Cryptographic pairings and applications

Cryptographic pairings are bilinear, non-degenerate and computable maps, defined on elliptic or hyperelliptic curves. The primary application of a pairing is to enable tri-partite key exchange, but there are other useful and unique applications. Pairings can also be used for the remote secure authentication of authorized users using short space-efficient digital signatures. Pairings also have an important role in realizing ID-based cryptography, which uses a user's ID as a compact and user-friendly public encryption key. In this talk, we first introduce properties and some examples of pairings. We then discuss applications of pairings mentioned above.

OCTAVIAN MITRIA, University of Western Ontario
Open Whitney umbrellas are locally polynomially convex

The notion of polynomial convexity of compact subsets of the Euclidean complex space $\mathbb{C}^n$ plays a fundamental role in the general theory of approximation of continuous functions, uncovering deep connections to topology, Banach algebras, symplectic geometry, and other areas of mathematics. In this talk, we show that every totally real smooth surface in $\mathbb{C}^2$, with a singularity of the open Whitney umbrella type at the origin, is locally polynomially convex near the singularity. The motivation comes from the work of Givental who proved that for any compact real surface S there exists a smooth mapping of S into a symplectic manifold which is a Lagrangian embedding, except at a finite set of singular points. Moreover, these singularities can only be of the following two types: a transverse double self intersection, or an open Whitney umbrella. This is joint work with my adviser, R. Shafikov.

KENTO OSUGA, University of Alberta
Supereigenvalue Models and Topological Recursion

It is an open question how to construct intersection theory on moduli spaces of super-Riemann surfaces. Since there is a well known relation between expectation values of Hermitian matrix models and intersection numbers on moduli spaces of Riemann surfaces, a potential approach to this mystery is to study supereigenvalue models, which are supersymmetric generalizations of Hermitian matrix models. About ten years ago, Eynard and Orantin showed that expectation values of Hermitian matrix models can be nicely reconstructed recursively via a formalism now called topological recursion. In this talk I will explain that, perhaps surprisingly, the Eynard-Orantin topological recursion can also be used to calculate expectation values of supereigenvalue models. It remains to be seen whether these new results help further our understanding of moduli spaces of super-Riemann surfaces.

SIDDHI PATHAK, Queen's University
On a Conjecture of Livingston

In the early 1960s, Erdos conjectured that the L-series attached to periodic arithmetical functions with period q, taking the value 0 on multiples of q and values in 1, -1 otherwise, do not vanish at $s = 1$. In an attempt to resolve this conjecture, Livingston predicted the linear independence of logarithm of certain algebraic numbers. In this talk, we outline recent work settling Livingston's conjecture.

VICTOR I. BRAVO REYNA, UNAM, CIMAT-Merida
Electromagnetism on supersymmetric structures

The objective of this talk is to present Dirac-like operators which are compatible with orthogonal and symplectic geometry, simultaneously. In particular, on the conformal flat homogeneous model of 4-dimensional Minkoski space we are introducing a operator that contains Maxwell's equations on a supersymmetric enviroment.

YURIJ SALMANIW, McMaster University
Bounded Solutions to a Singular Parabolic System

Recently, authors have studied singular elliptic systems, such as

$$\begin{cases} -\Delta u = \frac{1}{u^p} + \frac{1}{v^q} \\ -\Delta v = \frac{1}{u^r} + \frac{1}{v^s}, & x \in \Omega, \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega, \end{cases}$$

in a smooth, bounded domain $\Omega \subset \mathbb{R}^N$. Such problems arise in the study of enzyme kinetics, chemical reactions that are catalyzed by enzymes, as well as some applications to physics used to describe the gravitational potential of self-gravitating, spherically symmetric, polytropic fluid.

This presentation concerns the recently submitted joint work with S. Chen and R. Xu on the following singular parabolic system

$$\begin{cases} u_t = \Delta u + \frac{f(x)}{v^p} \\ v_t = \Delta v + \frac{g(x)}{u^q}, & t > 0, x \in \Omega, \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega, \end{cases}$$

a natural extension of the elliptic problem found above. Under suitable conditions for real valued constants $p, q > 0$ and the functions $f(x)$, $g(x)$, existence of weak and classical solutions are obtained using a powerful functional method. Such equations are interesting due to their singular nature on the boundary $\partial\Omega$. Applications of classical methods are unsuccessful, so alternative methods must be utilized. Through obtaining bounds for functions combining the solutions u, v, and ϕ, the first normalized eigenfunction of $-\Delta$, we obtain uniform L^p bounds for $u_\varepsilon, v_\varepsilon$ in a related perturbation problem. Then, we use Sobolev embedding theorem to obtain classical solutions.

NANCY WALLACE, Université du Québec à Montréal (UQAM)
Une généralisation de l'opérateur ∇. A generalization of the ∇ operator. (Talk in French slides in English)

Introduits en 1988, par Ian G. Macdonald, les polynômes de Macdonald ont, depuis, fait l'objet de beaucoup de recherche. Dans cet exposé nous discuterons d'opérateurs linéaires à coefficients dans $\mathbb{Q}[q, q^{-1}, t, t^{-1}]$ ayant les polynômes de Macdonald comme vecteurs propres. Il n'est pas nécessaire de connaitre les polynômes de Macdonald pour comprendre l'exposé, il sera fait du point de vue des opérateurs en axiomisant les propriétés induites par la base de vecteurs propres. Plus précisément nous introduirons un ensemble d'opérateurs dits monomiaux généralisant l'opérateur ∇ introduit par F.Bergeron et A.Garcia dans Science Fiction and Macdonald's Polynomials. Nous discuterons des opérateurs monomiaux ayant une interprétation combinatoire, nous en ferons une caractérisation, ainsi qu'une caractérisation de ∇. Si le temps nous le permet, nous verrons comment ceci peut nous aider à résoudre une conjecture et certains problèmes.

Talk on a certain class of Macdonald eigenfuctions.