Control of Partial Differential Equations
Contrôle des équations aux dérivées partielles

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ASSIA BENABDALLAH, I2M, Marseille-Aix-Marseille University, CNRS, Centrale

New phenomena for the null controllability of parabolic systems

One of the main goal in control theory is to drive the state of the system to a given configuration using a control that act through a source term located inside the domain or through a boundary condition.

The reference works for the control of linear parabolic problems are due to H.O. Fattorini and D.L. Russell in the 70's for the one dimensional case and to A.V. Fursikov, O.Yu. Imanuvilov on one side and G. Lebeau, L. Robbiano on the other side both in the 90's for the multi-dimensional case. They established null-controllability of heat equations with distributed or boundary controls in any time and for any control domain.

The aim of this talk is to give an overview on the recent results on the controllability of parabolic systems. Through simple examples, I will show that new phenomena appear as minimal time of control, dependance on the location of the control.

BIANCA CALSAVARA, University of Campinas

Local exact controllability of two-phase field solidification systems with few controls

In this work it is analyzed a control problem with a reduced number of controls for a phase field system modeling a solidification process of materials that allow two different types of crystallization and the flow of material in the non-solid regions. In this system we have three Allen-Cahn equations describing the phase field functions coupled to modified Navier-Stokes system and a heat equation for the temperature. It is proved that this system is locally exactly controllable to suitable homogeneous trajectories with controls acting only on the velocity field and heat equations. One of the difficulties of this work is that the three phase field equations are controlled by the velocity and temperature functions, but the coupling is multiplicative in the mentioned equations.

MARCELO MOREIRA CAVALCANTI, Universidade Estadual de Maringa

UNIFORM DECAY RATE ESTIMATES FOR THE SEMILINEAR WAVE EQUATION IN INHOMOGENEOUS MEDIA WITH LOCALLY DISTRIBUTED NONLINEAR DAMPING

We consider the semilinear wave equation posed in an inhomogeneous medium Ω with smooth boundary $\partial\Omega$ subject to a non linear damping distributed around a neighborhood ω of the boundary according to the Geometric Control Condition. We show that the energy of the wave equation goes uniformly to zero for all initial data of finite energy phase-space.

VALERIA CAVALCANTI, Universidade Estadual de Maringa

Exponential stability for the wave model with localized memory

We discuss the asymptotic stability as well as the well-posedness of a damped wave equation subject to a locally distributed viscoelastic effect. The results presented in this talk have been obtained in collaboration with M. Cavalcanti, M. A. Jorge Silva and A. Y. de Souza Franco.

EDUARDO CERPA, Universidad Técnica Federico Santa María

On the stability of some PDE-ODE systems involving the wave equation

Systems coupling an ordinary differential equation (ODE) with a wave through its boundary data are considered in this talk. The main focus is put on the role of different time scales on the stability of the coupled system. A fast wave equation coupled to a slow ODE is proven to be stable if each subsystem is stable. However, we show examples of stable subsystems generating an unstable full system when coupling a slow wave equation to a fast ODE. This is a joint work with Christophe Prieur (Grenoble, France).

MICHEL DELFOUR, Centre de recherches mathématiques, Université de Montréal
Shape and Topological Derivatives via One Sided Differentiation of Lagrangian Functionals

The generic notions of shape and topological derivatives have proven to be both pertinent and useful from the theoretical and numerical points of view. The shape derivative is a differential while the topological derivative often obtained by the method of matched and compound expansions is only a semidiferential (one sided directional derivative). This arises from the fact that the tangent space to the underlying metric spaces of geometries is only a cone. In my recent work the definition of a topological derivative is extended to perturbations obtained by creating holes around curves, surfaces, and microstructures by using the d-dimensional Minkowski content and sets of positive reach. In that context, the Hadamard semidifferential that retains the advantages of the standard differential calculus including the chain rule and the fact that semiconvex functions are Hadamard semidifferentiable is a natural notion to study the semidifferentiability of objective functions with respect to the sets/geometries that belong to complete non-linear non-convex metric spaces.

An important advantage for state constrained objective functions is that theorems on the one-sided differentiation of minimax of Lagrangians can be used to get the semidifferential. For instance, a standard approach to the minimization of a state constrained objective function in Control/ Shape Optimization problems is to consider the minimax of the associated Lagrangian. By using the new notion of averaged adjoint introduced by Sturm and new conditions by Delfour-Sturm, the minimax problem need not be related to a saddle point: non-convex objective functions and non-linear state equations can be directly considered.

LUDOVICK GAGNON, Universite Cote d'Azur
Sufficient conditions for the boundary controllability of the wave equation with transmission conditions

Let $T > 0$ and $\Omega, \Omega_2 \subset \mathbf{R}^2$ be open, connected and strictly convex domains of smooth boundaries $\partial\Omega, \partial\Omega_2$ and such that $\overline{\Omega_2} \subset \Omega$. Let $\Omega_1 := \Omega \setminus \overline{\Omega_2}$. We define the wave equation with transmission conditions by

$$\begin{cases} (\partial_{tt} - c_i^2\Delta)u^i(x,t) = 0, & (x,t) \in \Omega_i \times (0,T), \\ u^1(x,t) = u^2(x,t), & (x,t) \in \partial\Omega_2 \times (0,T), \\ c_1^2\partial_{n_2}u^1(x,t) = -c_2^2\partial_{n_2}u^2(x,t), & (x,t) \in \partial\Omega_2 \times (0,T), \\ u^1(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u^i(x,0) = u_0^i(x), u_t^i(x,0) = u_1^i(x), & (x,t) \in \Omega_i \times (0,T), \end{cases}$$

where $c_i > 0, i = 1,2$, n_2 is the outward unit normal of Ω_2 and $(u_0^1\mathbf{1}_{\Omega_1} + u_0^2\mathbf{1}_{\Omega_2}, u_1^1\mathbf{1}_{\Omega_1} + u_1^2\mathbf{1}_{\Omega_2}) \in H_0^1(\Omega) \times L^2(\Omega)$. We consider the boundary observability of the wave equation with transmission conditions : for $\Gamma \subset \partial\Omega$, does there exist $T > 0$ and $C_T > 0$ such that

$$E(u^1,u^2)(t) \le C_T \int_0^T \int_\Gamma \left|\partial_n u^1(x,t)\right|^2 \mathrm{dxdt}$$

holds? It is known that the observability fails to hold if $c_2 < c_1$ since there are rays of the optic geometry that cannot leave Ω_2. Thus, consider the case where $c_2 > c_1$, the case $c_2 = c_1$ reducing to the observability of the classical wave equation.

We prove the following. Let $x_0 \in \mathbf{R}^2 \setminus \overline{\Omega}$ and $\Gamma(x_0) := \{x \in \partial\Omega \,|\, (x - x_0).n(x) > 0\}$. If the flow induced by the boundaries $\partial\Omega_2$ and $\partial\Omega \setminus \Gamma(x_0)$ is dispersive, then for any $c_2 > c_1$, there exist $T > 0$ such that the observability holds. Otherwise, there exists $c^* > 1$ such that for every $c^* > c_2/c_1$, there exists $T > 0$ such that the observability holds.

SCOTT HANSEN, Iowa State University
Boundary controllability of Schrodinger and beam equation with an internal point mass

I'll describe some boundary control results for a Schrödinger equation on the domain $(-1,0)\cup(0,1)$ with a singular transmission condition at $x=0$:

$$\begin{cases} u_t + iu_{xx} = 0, & x \in (-1,0)\cup(0,1),\ t>0 \\ u(0^-,t) = u(0+,t) & t>0 \\ \frac{d}{dt}u(0,t) + i[u_x(0^+,t) - u_x(0^-,t)] = 0 & t>0 \\ u(-1,t) = 0, & t>0 \end{cases}$$

with either Dirichlet control : $u(1,t) = f(t)$, or Neumann control: $u_x(1,t) = f(t)$. In the Neumann case, we find results analogous to known results for the wave equation, in which exact controllability holds on a space with differing regularities on each side of the interface. This is not the case however in the case of Dirichlet control, where the controllability space is the same on each side. Some general results (for non-symmetric domains) will also be described. Related null-controllability results for the Euler-Bernoulli equation will also be discussed.

ALBERTO MERCADO, Universidad Técnica Federico Santa María
Controllability of wave equations in heterogeneous media

We present a controllability problem for semilinear wave equations with discontinuous main coefficient:

$$u_{tt} - div(a\nabla u) + g(u) = f \tag{1}$$

These equations apear as a model of wave propagation in heterogeneous media. We deal with the case of locally constant main coefficients with a jump in the boundary of a interior subdomain Ω_1:

$$a(x) = \begin{cases} a_1 & x \in \Omega_1 \\ a_2 & x \in \Omega \setminus \overline{\Omega}_1 \end{cases}$$

with $a_j > 0$ for $j = 1,2$.

Then we consider the equation as a pair of wave equation with constant coefficients a_1, a_2, coupled with the so-called transmission conditions

$$\begin{cases} u_1 = u_2 & \text{on } \partial\Omega_1 \\ a_1 \frac{\partial u_1}{\partial n_1} + a_2 \frac{\partial u_2}{\partial n_2} = 0 & \text{on } \partial\Omega_1. \end{cases} \tag{2}$$

We will show controllability results when the Ω_1 is convex and some generalizations. The main tool is the use of adequate Carleman estimates. Also, we will present some related problems regarding models for wave propagation in beams and some inverse problems.

CRISTHIAN MONTOYA, Universidad Nacional Autónoma de México
Robust Stackelberg controllability for the Navier-Stokes equations

In this talk we deal a robust Stackelberg strategy for the Navier-Stokes system. The scheme is based by considering a robust control problem for the "follower player" and its associated disturbance function. Secondly, we use the notion of Stackelberg optimization (which is associated to the "leader player") in order to deduce a local null controllability result for the Navier-Stokes system. In collaboration with Luz de Teresa.

KIRSTEN MORRIS, University of Waterloo
Optimal Sensor Design in Estimation

Estimator performance depends on the sensors as well as the estimator design. This is particularly an issue for systems modeled by partial differential equations. For these systems, the location of the sensors can often be chosen. There is also generally a variety of sensor types available that measure different aspects of the state. Furthermore, the development of smart materials means that in some cases, the shape of the sensor can also be designed. It is reasonable to use the same criteria for sensor design and location as for the estimator design. The solution to an operator Riccati equation minimizes the steady-state error variance. This extends a result previously known for finite-dimensional systems. The trace of the Riccati operator is thus a reasonable cost function. A framework for calculation of the best sensor locations using approximations is established. This allows the question of sensor choice to be investigated since each sensor type can be assumed to be optimally located. The problem of optimal sensor shape is mathematically complex. It is first stated formally, and then it is shown to be well-posed and to possess optimal solutions under certain conditions. The results are illustrated by several examples.

ADEMIR PAZOTO, Federal University of Rio de Janeiro (UFRJ)
Control and Stabilization for a coupled system of linear Benjamin-Bona-Mahony type equations

We analyze the boundary controllability and stabilization properties of the linearized Boussinesq system of BBM-BBM type introduced by J. Bona, M. Chen and J.-C. Saut as a model for the motion of small amplitude long waves on the surface of an ideal fluid.

By means of a careful spectral analysis of the operator associated with the state equations we show that the system is approximately controllable but not spectrally controllable. This means that no finite linear nontrivial combination of eigenvectors can be driven to zero in finite time.

We also propose several dissipation mechanisms leading to systems for which one has both the existence of solutions and a nonincreasing norm. It is shown that all the trajectories are attracted by the origin provided that the Unique Continuation Property holds.

IVONNE RIVAS, Universidad del Valle
Stabilization for a type of uncontrollable systems

In this talk, we will study the problem of stabilization of a kind of system in finite dimension, which has some uncontrollable directions, finding an explicit feedback law that stabilizes the system to zero. Some examples will be showed. Later, we extended this problem to an infinity dimensional control problem, more specifically the Korteweg-de Vries equation on the interval $[0, L]$ with a length that makes the lineal system uncontrollable, obtaining the stabilization of the system to zero.

SHUXIA TANG, University of Waterloo
Sensor Design for Distributed Parameter Systems

The development of smart materials makes it feasible to optimize sensor shapes. Optimization over shape is challenging though since the admissible set is infinite-dimensional. Kalman filters are optimal in the sense that they minimize the estimation error variance for given sensors. They are thus popular state estimators for both lumped and distributed parameter systems. Choosing the estimation error variance as the optimization criterion, conditions have been obtained that guarantee well-posedness of the optimal sensor design problem as well as continuity of the cost with respect to sensor design. A framework is constructed using finite-dimensional approximations for calculation of the optimal shape. Sufficient conditions are provided for the finite-dimensional optimal sensor configurations to converge to the infinite-dimensional optimal estimation performance.

BINGYU ZHANG, University of Cincinnati
General Boundary Value Problems of the Kortweg-de Vries Equation on a Bounded Domain

In this talk we consider the initial boundary value problem of the Korteweg-de Vries equation posed on a finite interval

$$u_t + u_x + u_{xxx} + uu_x = 0, \qquad u(x,0) = \phi(x), \qquad 0 < x < L,\ t > 0 \tag{3}$$

subject to the nonhomogeneous boundary conditions,

$$B_1 u = h_1(t), \qquad B_2 u = h_2(t), \qquad B_3 u = h_3(t) \qquad t > 0 \tag{4}$$

where

$$B_i u = \sum_{j=0}^{2} \left(a_{ij} \partial_x^j u(0,t) + b_{ij} \partial_x^j u(L,t) \right), \qquad i = 1, 2, 3,$$

and $a_{ij},\ b_{ij}$ $(j, i = 0, 1, 2, 3)$ are real constants. Under some general assumptions imposed on the coefficients $a_{ij},\ b_{ij}$, $j, i = 0, 1, 2, 3$, the IBVPs (3)-(4) is shown to be locally well-posed in the space $H^s(0, L)$ for any $s \geq 0$ with $\phi \in H^s(0, L)$ and boundary values $h_j, j = 1, 2, 3$ belonging to some appropriate spaces with optimal regularity. This a joint work with R. A. Capistrano-Filho of Universidade Federal de Pernambuco and Shuming Sun of Virginia Tech.