Contributed Papers
Communications libres
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ABDULLAHI ADEM, North-West University
Multiple wave solutions and conservation laws of the Date-Jimbo-Kashiwara-Miwa (DJKM) equation via symbolic computation

In this talk, we present soliton solutions and conservation laws for the DJKM equation with the aid of symbolic computation. The soliton solutions of the DJKM equation are constructed by using the multiple exp-function method, which is a generalization of Hirota's perturbation scheme. The solu- tions obtained involve generic phase shifts and wave frequencies. Furthermore, in nitely many conservation laws are derived by using the multiplier method which is an indicator of the integrability of the underlying equation.

PEDRO PABLO CARDENAS ALZATE, Universidad Tecnológica de Pereira
The Zhou's method for solving delay differential equations applied to biological models

In this work, we apply the Zhou's method or Differential Transfor- mation Method (DTM) for solving some models that arises in biological sciences, which are nonlinear delay differential equations. The efficiency of DTM is illustrated by investigating the convergence results on numerical models that show the reliability and accuracy of this method.

ERIC JOSE AVILA, Universidad Autonoma de Yucatan
Global dynamics of a periodic SEIRS model with general incidence rate

We consider a family of periodic SEIRS epidemic models with a fairly general incidence rate, we will show that the basic reproduction number determines the global dynamics of the models and it is a threshold parameter for persistence. We estimate the basic reproduction number and we provide numerical simulations to illustrate our findings.

VLADISLAV BUKSHTYNOV, Florida Institute of Technology
Optimal Reconstruction of Constitutive Relations for Porous Media Flows

Comprehensive full-physics models for flow in porous media typically involve convection-diffusion partial differential equations whose parameters are unknown and have to be reconstructed from experimental data. Quite often these unknown parameters are coefficients represented by space-dependent, sometimes correlated, functions, e.g. porosity, permeability, transmissibility, etc. However, special complexity is seen when the reconstructed properties are considered as state-dependent parameters, e.g. the relative permeability coefficients k_{rp}. Modern petroleum reservoir simulators still use simplified approximations of k_{rp} as single variable functions of p-phase saturation s_p given in the form of tables or simple analytical expressions. This form is hardly reliable in modern engineering applications used, e.g., for enhanced oil recovery, carbon storage, modeling thermal and capillary pressure relations. Thus, the main focus of our research is on developing a novel mathematical concept for building new models where k_{rp} are approximated by multi-variable functions of fluid parameters, namely phase saturations s_p and temperature T. Reconstruction of such complicated dependencies requires advanced mathematical and optimization tools to enhance the efficiency of existing engineering procedures with a new computational framework generalized for use in various earth science applications.

YUAN-JEN CHIANG, University of Mary Washington
Leaf-wise Harmonic Maps of Manifolds with 2-dimensional Foliations

In the 1980s, A. Connes [Proc. of Symp. in Pure Math, AMS, 1982] and E. Ghys [J. Func. Anal., 1988] proved the Gauss-Bonnet type theorem for compact manifolds with 2-dimensional foliations. In this paper, we derive the expressions of harmonic non $\pm$ holomorphic maps of Riemann surfaces. We study the relationship between leaf-wise harmonic maps and harmonic maps. We investigate the Gauss-Bonnet type theorem for leaf-wise harmonic maps between manifolds with 2-dimensional foliations, which generalize the results of Connes and Ghys. This paper has recently appeared in the Bulletin of the Institute of Mathematics, Academia, Sinica.

BUTHINAH BIN DEHAISH, King Abdullaziz University
Fixed Point Theorem for monotone Lipschitzian mappings

Among this talk we will consider a new class of Lipschitzian mappings which are monotone and then we will discuss some fixed point theorems for these mappings.

RYAD GHANAM, Virginia Commonwealth University in Qatar
Non-Solvable subalgebras of gl(4,R)

In this talk, I will present all the simple, then semi-simple, subalgebras of gl(4, R). Each such semi-simple subalgebra acts by commutator on gl(4, R). In each case the invariant subspaces are found and the results used to determine all possible subalgebras of gl(4, R) that are not solvable

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NAZISH IFTIKHAR, National University of Computer and Emerging Sciences, Lahore Campus, Pakistan.
Classifying Robertson-Walker scale factor using Noether's approach

The universe can be depicted in the best way by using Friedmann-Robertson-Walker (FRW) models. FRW models of the universe are considered to have properties like homogeneity and isotropy. The universe is continuously expanding which can be represented by considering Robertson-Walker scale factor. Robertson-Walker scale factor is the function of time 't'. The scale factor is useful to define red shift and the Hubble parameter. The Hubble parameter gives information about the evolution of the universe and is also useful in calculating the age of the universe. In present research work, Noether's approach was applied to classify FRW spacetime. The spacetime was considered for three types of universe i.e. closed, open, and flat. For closed, open and flat universe, curvature parameter 'k' was -1, 1, and 0 respectively. Different values of Robertson-Walker scale factor were considered which gave the nontrivial symmetries. By using Noether equation and Perturbed Lagrangian an over-determined system of partial differential equations were obtained. For the closed, open and flat universe, maximal and minimal set of Noether operators were acquired. For every Noether operator, the corresponding energy type first integral of motion was calculated.

LAHCEN LAAYOUNI, Al Akhawayn University
On the efficiency of the Algebraic Optimized Schwarz Methods

In this study we investigate the efficiency of the Algebraic Optimized Schwarz Methods (AOSM) in solving large-scale linear systems. The AOSM used as preconditionners in solving linear systems converge in two iterations for a decomposition with two sub-domains using optimal transmission blocks. These blocks require the inverse of large sub-matrices of the original matrix of the linear system. In this paper we are interested in approximating the transmission blocks with adequate approximations. Numerical comparisons will be presented for different types of problems.

This is joint work with M. Gander and D. Szyld.

EUGEN MANDRESCU, Holon Institute of Technology, Israel
Shedding vertices and well-covered graphs

A set S of vertices in a graph G is *independent* if no two vertices from S are adjacent. If all maximal independent sets are of the same cardinality, then G is *well-covered* (or *unmixed*) (Plummer, 1970). G belongs to class $\mathbf{W}_2$ if every 2 disjoint

independent sets are included in 2 disjoint maximum independent sets (Staples, 1975). There are deep interactions between shellability, vertex decomposability and well-coveredness (Castrillón, Cruz, Reyes, 2016).

Let $v \in V(G)$ and $N(v)$ be its neighborhood. If for every independent set S of $G-(N(v) \cup \{v\})$, there is some $u \in N(v)$ such that $S \cup \{u\}$ is independent, then v is a *shedding vertex* of G (Woodroofe, 2009). Let $Shed(G)$ denote the set of all shedding vertices. Clearly, no isolated vertex is shedding, and no graph in $\mathbf{W}_2$ has isolated vertices.

In this talk, we show that deleting a shedding vertex does not change the maximum size of a maximal independent set including a given independent set. Specifically, for well-covered graphs, it means that a non-isolated vertex $v \in Shed(G)$ if and only if $G-v$ is well-covered. Thus G belongs to class $\mathbf{W_2}$ if and only if $Shed(G) = V(G)$.

There exist well-covered graphs without shedding vertices; e.g., C_7. On the other hand, there are non-well-covered graphs with $Shed(G) = V(G)$.

Problem 1. Find all well-covered graphs having no shedding vertices.

Problem 2. Find all graphs having $Shed(G) = V(G)$.

IMRAN NAEEM, Lahore University of Management sciences (LUMS), Pakistan
A new approach to construct first integrals and closed-form solutions of dynamical systems for epidemics

A new class of non-standard Hamiltonian known as the "artificial Hamiltonian" is introduced which results in an artificial Hamiltonian system of first-order ordinary differential equations (ODEs). The notion of an artificial Hamiltonian is developed for the systems of dynamical systems of ODEs. Also, it is shown that every system of second-order ODEs can be expressed as an artificial Hamiltonian system of first-order ODEs. The newly developed notion of an artificial Hamiltonian system gives a new way to solve the dynamical systems of first-order ODEs or systems of second-order ODEs which can be expressed as an artificial Hamiltonian system by utilizing the known techniques applicable to the non-standard Hamiltonian systems. We employed this proposed notion to solve dynamical systems of first-order ODEs arising in epidemics.

REHANA NAZ, Lahore School of Economics, Pakistan
The first integrals and closed-form solutions of optimal control problems

The Pontrygin's maximum principle (Pontryagin, 1987) provides the necessary conditions for the optimum in the optimal control problems in terms of variables time t, state variables q^i, costate variables p_i and control variables u_i. One can eliminate the control variables in terms of state and co-state variables which reduces the conditions of Pontrygin's maximum principle to following non-standard Hamiltonian system:

$$\dot{q}^i = \frac{\partial H}{\partial p_i}, \dot{p}^i = -\frac{\partial H}{\partial q_i} + \Omega^i(t, q^i, p_i.)$$

This type of non-standard Hamiltonian system arises widely in optimal control problems in different fields of the applied mathematics. A mechanical system with non-holonomic nonlinear constraints and non-potential generalized forces results in a non-standard Hamiltonian system. In optimal control problems of economic growth theory involving a non-zero discount factor these type of system arise and are known as a current value Hamiltonian systems. It is proposed how to modify the partial Hamiltonian approach proposed earlier for the current value Hamiltonian systems arising in economic growth theory Naz et al 2014 in order to apply it to the epidemics, mechanics and other areas as well. To show the effective of the approach developed here, it is utilized to construct the first integrals and closed form solutions of some models from real world. Moreover, the essential aspects of infectious diseases spread are uncovered and polices are provided to public health decision makers to compare and implement different control programs. For the Economic growth model some policies are provided to the government in order to have a sustainable growth.

KARIM SAMEI, Bu Ali Sina University, Hamedan, Iran.
Singleton Bounds for R-additive Codes

Shiromoto (Linear algebra Applic 295 (1999) 191-200) obtained the basic exact sequence for the Lee and Euclidean weights of linear codes over $\mathbb{Z}_\ell$ and as an application, he found the Singleton Bounds for linear codes over $\mathbb{Z}_\ell$ with respect to Lee and Euclidean weights. Huffman (Adv. Math. Commun 7 (3) (2013) 349-378) obtained the Singleton Bound for $\mathbb{F}_q$-linear $\mathbb{F}_{q^t}$-codes with respect to Hamming weight?. ?Recently the theory of $\mathbb{F}_q$-linear $\mathbb{F}_{q^t}$-codes were generalized to R-additive codes over R-algebras by Samei and Mahmoudi. In this paper, we generalize Shiromoto's results for linear codes over $\mathbb{Z}_\ell$ to R-additive codes. As an application, when R is a chain ring, we obtain the Singleton Bounds for R-additive codes over free R-algebras. Among other results, the Singleton Bounds for additive codes over Galois rings are given.

DOMINGO TARZIA, CONICET and Universidad Austral

The one-phase Stefan problem with a latent heat of fusion depending of the position of the free boundary and its velocity

From the one-dimensional consolidation of fine-grained soils with threshold gradient, it can be derived a special type of Stefan problems where the seepage front, due to the presence of this threshold gradient, exhibits the features of a moving boundary. In this kind of problems, in contrast with the classical Stefan problem, the latent heat depends inversely with the rate of change of the seepage front (e.g. Zhou-Bu-Lu, Int. J. Numerical and Analytical Methods in Geomechanics, 37 (2013), 2825-2832). A one-phase Stefan problem with a latent heat that not only depends on the rate of change of the free boundary but also on its position is studied. The aim of this analysis is to extend prior results, finding an analytical solution that recovers, by specifying some parameters, the solutions already examined in the literature regarding Stefan problems with variable latent heat. Moreover, we also consider different boundary conditions at the fixed face. This is a joint paper with Julieta Bollati (CONICET and Universidad Austral).

STEFAN VELDSMAN, Nelson Mandela University

Generalized complex numbers over near-fields

In the early 20th century, Dickson (1905) investigated the redundancy or not of the field axioms. By a clever disturbance of the multiplication of a field, he demonstrated the existence of an algebraic structure fulfilling all the requirements of a field except one of the distributive axioms. These structures are known as Dickson near-fields, but there are many near-fields not of this type. Almost immediately near-fields were shown to be not just an algebraic curiosity. Veblen and Wedderburn (1907) showed that near-fields are exactly the algebraic structures required for coordinization of geometries that lead to non-Desarguesian planes. In a monumental paper, Zassenhaus (1935/6) showed that all finite near-fields are Dickson near-fields except for 7 strays. There are many other applications of near-fields and the more general near-rings became an important and useful area of investigation with its own concerns and problems catering for non-linear algebraic systems.

The construction of the complex numbers over the reals has been generalized in many ways leading to the 2-dimensional elliptical complex numbers (= complex numbers) and the parabolic and hyperbolic complex numbers. These can be extended to higher dimensions and using an arbitrary ring as the base ring.

It is possible to define matrices and polynomials over near-rings. Using these, one can construct generalized complex numbers over a near-field. In this talk, this construction will be formalized. We also report on properties of this algebraic structure and highlight similarities and differences with its motivating example; the usual complex numbers over the real field.

CHUANG XU, University of Alberta

Best finite constrained approximations of one-dimensional probabilities

This paper studies best finitely supported approximations of one-dimensional probability measures with respect to the L^r-Kantorovich (or transport) distance, where either the locations or the weights of the approximations' atoms are prescribed. Necessary and sufficient optimality conditions are established, and the rate of convergence (as the number of atoms goes to infinity) is discussed. Special attention is given to the case of best uniform approximations (i.e., all atoms having equal weight). The elementary approach is based on best approximations of (monotone) L^r-functions by step functions, which is different from, and naturally complementary to, the classical Voronoi partition approach. This is a joint work with Dr. Arno Berger.