Combinatorial Commutative Algebra
Algèbre commutative combinatoire
(Org: **Sara Faridi** (Dalhousie University), **Susan Morey** (Texas State University) and/et **Rafael Villarreal** (Centro de Investigacion y de Estudios))

JENNIFER BIERMANN, Hobart and William Smith Colleges
Colorings of simplicial complexes and vertex decomposability

In attempting to understand how combinatorial modifications alter algebraic properties of monomial ideals, several authors have investigated the process of adding "whiskers" to graphs. In this paper, we study a similar construction to build a simplicial complex Δ_χ from a coloring χ of a subset of the vertices of Δ, and give necessary and sufficient conditions for this construction to produce vertex decomposable simplicial complexes. We apply this work to strengthen and give new proofs about sequentially Cohen-Macaulay edge ideals of graphs.

SUSAN COOPER, North Dakota State University
The Waldschmidt Constant For Monomial Ideals

The Waldschmidt constant is a limit which was first introduced as a way to estimate the lowest degree of a hypersurface vanishing at all the points of a variety to a given order. This special limit can be used to find failure of containments between symbolic and regular powers of a homogeneous ideal. However, this useful limit is challenging to compute. We will give some interpretations of the Waldschmidt constant of a monomial ideal which allow us to determine this useful limit in a number of cases. This is joint work from two projects: the first with R. Embree, H. T. Hà, and A. Hoefel and the second with C. Bocci, E. Guardo, B. Harbourne, M. Janssen, U. Nagel, A. Seceleanu, A. Van Tuyl, and T. Vu.

ANTON DOCHTERMANN, Texas State University
Trees, skeleta, and combinatorics of monomial chip-firing ideals

For any graph G, one can construct the 'G-parking function ideal' M_G, a monomial ideal whose standard monomials are in bijection with the spanning trees of G (and hence connect to various other combinatorial objects). Postnikov and Shapiro studied the ideals M_G in connection with power ideals and other deformations of 'monotone monomial ideals', and constructed minimal resolutions for certain classes. Minimal cellular resolutions of M_G for arbitrary G were later described by Dochtermann and Sanyal.

The ideals M_G are also strongly related to 'chip-firing' on the graph G, a dynamical system on the vertices governed by the Laplacian matrix. Motivated by these notions we study certain 'skeleta' of the ideals M_G, generated by certain subsets of the vertices of G. For some large classes we construct minimal resolutions and describe monomial bases. These constructions involve a number of combinatorial gadgets including tropical hyperplanes, the 'signless' Laplacian, and (new?) enumerations of Cayley trees.

ART DUVAL, University of Texas at El Paso
A non-partitionable Cohen-Macaulay simplicial complex, and implications for Stanley depth

Cohen-Macaulay simplicial complexes are those whose face-ring is a Cohen-Macaulay ring; there is also a topological characterization. A simplicial complex is *partitionable* if its partially ordered set of faces can be partitioned into certain kinds of intervals. In 1979, Richard Stanley posed the following conjecture relating these ideas, which he later described as "a central combinatorial conjecture on Cohen-Macaulay complexes":

Conjecture: Every Cohen-Macaulay simplicial complex is partitionable.

We disprove this conjecture by constructing an explicit counterexample in three dimensions. Due to a result of Herzog, Jahan and Yassemi, our construction also disproves the well-known conjecture that the Stanley depth of a monomial ideal is always at least its depth. This is joint work with Bennet Goeckner, Carly Klivans, and Jeremy Martin.

CHRIS FRANCISCO, Oklahoma State University
Borel ideals with two Borel generators and Koszulness

Motivated by results of Conca and De Negri, we investigate the Koszul property for toric rings associated to Borel ideals with exactly two Borel generators.

MARYAM EHYA JAHROMI, Dalhousie University
Whitney's Theorem and Subideals of Monomial Ideals

The reconstruction of a graph from certain subgraphs has always been an interesting topic in graph theory and is still an open question. The vertex-deck (or simply the deck) of a graph G is the family of all its subgraphs which are obtained by removing exactly one vertex of G. Similarly we can define the edge-deck of a graph G.

The main question is whether one can uniquely determine a graph from its unlabeled vertex-deck. In 1964, Harary conjectured that any two graphs with at least four edges and the same edge-deck are isomorphic.

Long before that, in 1932 Whitney proved that if the line graphs of two simple graphs G and H are isomorphic, then G and H are also isomorphic except for the cases K_3 and $K_{1,3}$. Using this result, Hemminger proved that the edge reconstruction conjecture for graphs is equivalent to the vertex reconstruction conjecture for line graphs.

Trying to extend Whitney's theorem to hypergraphs, Berge introduced two hypergraphs $\mathcal{E}_p$ and $\mathcal{O}_p$ and proved that if two hypergraphs have isomorphic $(p-1)$-edge-decks then they are isomorphic only if they do not contain an $\mathcal{E}_p - \mathcal{O}_p$ pair. In 1987 Gardner proved the other direction under extra hypotheses.

In this talk we will discuss the ideal theory of hypergraphs with isomorphic $(p-1)$-edge-decks, using the results mentioned above.

KUEI-NUAN LIN, Penn State Greater Allegheny
Generalized Newton Complementary Duals of Monomial Ideals

Given a monomial ideal in a polynomial ring over a field, we define the generalized Newton complementary dual of the given ideal. The Newton complementary duals of monomial ideals were first introduced by Costa and Simis. We show good properties of such duals including linear quotients and isomorphisms between the special fiber rings. We construct the cellular free resolutions of duals of strongly stable ideals generated in the same degree. When the base ideal is generated in degree two, we provide an explicit description of cellular free resolution of the dual of a compatible generalized stable ideal. This is joint work with Katie Ansaldi and Yi-Huang Shen.

SONJA MAPES, University of Notre Dame
Computing projective dimension of monomial ideals via associated hypergraphs and lcm-lattices

Given a square-free monomial ideal I in a polynomial ring R over a field k, we would like to know the projective dimension of I. We recall the definition of the lcm-lattice of a monomial ideal introduced by Gasharov, Peeva and Welker, and the definition of the dual hypergraph of a square-free monomial ideal introduced by Kimura, Terai and Yoshida. Our work focuses on the relationship between the lcm-lattice and the dual hypergraph of a given square-free monomial ideal. We use the properties of lcm-lattice to find whether two different dual hypergraphs have the same projective dimension, and thus are able to extend some of the results by Lin and Mantero which compute the projective dimensions for ideals with certain hypergraphs. This is joint work with Kuei-Nuan Lin.

SARAH MAYES-TANG, University of Toronto
Betti tables of graded systems of ideals

Several patterns emerge in collections of Betti tables associated to the powers of a fixed ideal. For example, Wheildon and others demonstrated that the shapes of the nonzero entries of these tables eventually stabilize when the fixed ideal has generators of the same degree. In this talk, I will discuss patterns in the graded Betti numbers of these and other graded systems of ideals. In particular, I will describe ways in which the Betti tables may stabilize, and how different types of stabilization are reflected in the corresponding Boij-Söderberg decompositions.

JORGE NEVES, University of Coimbra
Regularity of the vanishing ideal over a parallel composition of paths

In 2011, Renteria, Simis and Villarreal introduced a new class of binomial ideals associated to graphs. One associates to a simple graph, G, the vanishing ideal of a subset of a projective space over a finite field, parameterized by the edges of G. The algebraic invariants of the ideal are then naturally related to the invariants of the series of evaluation codes obtained from the parameterized subset. In particular, we know that their regularity is the upper limit for the order of a nontrivial linear code in the series.

The focus of our talk will be rather the link between the regularity of the ideal and the combinatorial invariants of G. We will report on a recent joint work with A. Macchia, M. Vaz Pinto and R. Villarreal in which we compute the regularity in the case of a 2-connected graph given by a parallel composition of an arbitrary number of paths. Our results give evidence for the existence of a relation between the regularity of the ideal and a nontrivial invariant of G.

LUIS NUNEZ-BETANCOURT, CIMAT
Graph connectivity via homological invariants

Given a simple graph G, one can associate a ring R_G via monomial or binomial ideals. In this talk, we will discuss numbers that measure the connectivity of G and relate them to homological invariants of R_G. This talk includes joint work with Arindam Banerjee (Purdue) and Luis Pedro Montejano (CIMAT).

AUGUSTINE O'KEEFE, Connecticut College
Bounds on the regularity of toric ideals of graphs

Let G be a finite simple graph. We give a lower bound for the Castelnuovo-Mumford regularity of the toric ideal I_G associated to G in terms of the sizes and number of induced complete bipartite graphs in G. When G is a chordal bipartite graph, we find an upper bound for the regularity of I_G in terms of the size of the bipartition of G.

JAY SCHWEIG, Oklahoma State University
The type defect of a simplicial complex

We introduce an algebraic invariant of simplicial complexes called the "type defect." This invariant has close ties to graph chordality, as well as to various generalizations of the chordal property to simplicial complexes. The type defect is also linked to Cohen-Macaulay complexes, and can be used to study bi-Cohen-Macaulay complexes (CM complexes whose duals are CM as well). We will define new classes of complexes and graphs on which this invariant is especially well-behaved, and we will discuss how it changes when two complexes are glued together along a face. (This is joint work with Hailong Dao.)

SANDRA SPIROFF, University of Mississippi
Combinatorial aspects of intersection algebras

For $R = \mathsf{k}[x_1, \ldots, x_n]$ over a field k and ideals $I = (x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n})$ and $J = (x_1^{b_1} x_2^{b_2} \cdots x_n^{b_n})$ we obtain closed formulae in n, and the strings of nonnegative integers $\{a_1, \ldots, a_n\}, \{b_1, \ldots, b_n\}$, for various invariants of the intersection algebra. For a commutative Noetherian ring R, the intersection algebra of R with respect to I and J is $\mathcal{B}_R(I, J) = \bigoplus_{r,s\in\mathbb{N}} I^r \cap J^s$. This is joint work with Florian Enescu at Georgia State.

CARLOS VALENCIA-OLETA, Mathematics Department, Cinvestav-IPN
The combinatory of the arithmetical structures of the path

Given a graph $G = (V, E)$, an arithmetical structure (AS) on G is a pair $(\mathbf{d}, \mathbf{r}) \in \mathbb{N}_+^V \times \mathbb{N}_+^V$ such that $\mathbf{r}$ is primitive and

$$(\mathrm{diag}(\mathbf{d}) - A(G))\mathbf{r}^t = \mathbf{0}^t,$$

where $A(G)$ is the adjacency matrix of G. The concept of AS was introduced by Lorenzini as some intersection matrices that arise in the study of degenerating curves in algebraic geometry. To each AS can be associated a binomial ideal, which recently have been great interest.

In this talk we will give an explicit description of the AS of the path P_n with n vertices. Firstly, we will prove that all the AS on P_n can be obtained from the Laplacian of P_m (for $m \leq n$) using an edge subdivision process. Using this fact, ballot sequences and lattice paths we will get that the number of AS on P_n obtained from the Laplacian of P_m is the ballot number

$$b(n-2, n-m) = \frac{m-1}{n-1}\binom{2n-2-m}{n-2}.$$

Therefore the number of the AS on P_n is the Catalan number C_{n-1}.

On the other hand, using a concept of extended AS we will present a way to generate the AS of P_n from a single extended AS, which leads to establish a bijection between the AS on P_n and the triangulations of a polygon. Therefore the extended AS of P_n exhibit an invariance under rotations. Using this, we will get that the number of AS of P_n with its i entry equal to $n-k-1$ is equal to the ballot number $b(n-2, k)$.

ADAM VAN TUYL, McMaster University
Algebraic properties of circulant graphs

Let G be the circulant graph $C_n(S)$ with $S \subseteq \{1, 2, \ldots, \lfloor \frac{n}{2} \rfloor\}$. The family of circulant graphs includes both the cycles C_n and the cliques K_n. Let $I(G)$ denote the edge ideal of G in the ring $R = k[x_1, \ldots, x_n]$, and let $\mathrm{Ind}(G)$ denote the simplicial complex associated to $I(G)$ via the Stanley-Reisner correspondence. This talk will be a survey on what is known (and not known) about the algebraic properties of $I(G)$ and the topological properties of $\mathrm{Ind}(G)$.