Arithmetic Geometry and Related Topics
Géométrie arithmétique et sujets reliés
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ALEJANDRA ALVARADO, Eastern Illinois University
Arithmetic Progressions on Conic Sections

We view the set $\{(1,1),\,(5,25),\,(7,49)\}$ as a 3-term collection of rational points on the parabola $y=x^2$ whose y-coordinates form an arithmetic progression of perfect squares. In this talk we will provide a generalization to 3-term arithmetic progressions on arbitrary conic sections $\mathcal{C}$ with respect to a linear rational map $\ell:\mathcal{C}\to\mathbb{P}^1$.

ANTONIO CAFURE, UNGS, UBA, CONICET
Cyclotomic polinomials and linear algebra

Let n be an odd natural number and let p be an odd prime such that $p\nmid n$. In this talk, following the techniques of [1] and well known results about cyclotomic polynomials, we will show that the coefficients of the cyclotomic polynomial Φ_{np} can be computed as the unique solution of a linear system of equations $Tx=b$, where T is a semicirculant matrix involving coefficientes of Φ_n, and b is a vector whose entries are certain coefficients of Φ_n determined according to some congruences modulo p.

[1]. A. Cafure y E. Cesaratto. Irreducibility criteria for reciprocal polynomials and applications. Am. Math. Month. 124, No 1, 37–53.

MARÍA CHARA, Instituto de Matemática Aplicada del Litoral
Subtowers of towers of function fields

The function A(q), which measures how large the number of rational places in function fields (over finite fields) with respect to their genus can be, was introduced by Ihara in 1981. Few things are known about the exact value of this quantity and its importance appears, for example, in coding theory since good positive lower bounds for this function imply the existence of arbitrary long codes with asymptotically good parameters.

One way of obtaining non-trivial lower bound for Ihara's function is through the construction of asymptotically good towers of function fields over finite fields. An important contribution in this setting came from the hands of Garcia and Stichtenoth who exhibited explicit towers of function fields with asymptotically good limits using only basic facts of valuation and ramification theory. In general, it is not easy to determine whether a given explicit tower is asymptotically good or not. In some cases, the asymptotic behavior of the tower can be determined from the asymptotic behavior of a simpler subtower or supertower.

In this talk we will present a method to construct explicit and proper subtowers and supertowers of a given explicit tower. We will also give conditions to check if two apparently different equations define the same subtower or not. An interesting feature of our method is that it can be easily implemented in a computer so the search for explicit equations defining subtowers is rather simple.

RICARDO CONCEIÇÃO, Gettysburg College
Solutions of the Hurwitz-Markoff equation over polynomial rings

Let A and n be positive integers. The structure of the set of integral solutions of the equation

$$x_1^2+\cdots+x_n^2=Ax_1\cdots x_n \tag{1}$$

was first studied by Hurwitz, as a generalization of Markoff's equation (the case $n = A = 3$). Hurwitz showed that all integral solutions can be generated by the action of certain automorphisms of the hypersurface defined by (1) on finitely many solutions. Ever since, several authors have extended Hurwitz's work to the study of solutions of (1) over finite fields and number fields. Our goal is to discuss some progress made in understanding the solutions of (1) over the polynomial ring $k[t]$, where k is a field.

NATALIA GARCIA-FRITZ, University of Toronto
Curves of low genus and applications to Diophantine problems

In 2000, Paul Vojta solved the n-squares problem under the Bombieri-Lang conjecture, by explicitly finding all the curves of genus 0 or 1 on certain surfaces of general type related to this problem. In this talk I will sketch a refined and generalized version of the geometric method implicit in Vojta's work. I will also discuss new arithmetic applications conditional to the Bombieri-Lang conjecture in the case of number fields, and unconditional for function fields.

ANDREW HARDER, University of Miami
Calabi-Yau threefolds and modular curves

I'll discuss recent work which characterizes some of the types of K3 surfaces which arise as the generic fiber of a K3 fibration on a Calabi-Yau threefold. The proof uses basic facts about the modular curves associated to the groups $\Gamma_0(n)^+$. This is joint work with C.F. Doran and A. Thompson.

PIPER HARRON, University of Hawaii at Manoa
Equidistribution of Shapes of Number Fields of degree 3, 4, and 5

In her talk, Piper Harron will introduce the ideas that there are number fields, that number fields have shapes, and that for "random" number fields these shapes are everywhere you want them to be. This result is joint work with Manjul Bhargava and uses his counting methods which currently we only have for cubic, quartic, and quintic fields. She will sketch the proof of this result and leave the rest as an exercise for the audience. (Check your work by downloading her thesis!)

ROBERT HARRON, University of Hawai'i at Mānoa
Equidistribution of shapes of cubic fields of fixed quadratic resolvent

Building upon work of Bhargava, P. Harron, and Shnidman, I will discuss results on the distribution of shapes of cubic fields of fixed quadratic resolvent. The shapes depend on the trace zero form (that is the projection of the trace form to the trace zero space). For instance, I'll show that the shapes of complex cubic fields lie on the geodesic on the modular surface $\mathrm{SL}(2, \mathbf{Z})\backslash\mathfrak{H}$ determined by their trace zero form and that, in a fixed such geodesic, the shapes are equidistributed with respect to the natural hyperbolic measure. In the case of pure cubic fields (whose quadratic resolvent field is the third cyclotomic field), the corresponding geodesics have infinite length and the equidistribution must be considered in a regularized sense. That these geodesics are of infinite length provides a reason behind the different asymptotic growth rates of pure cubic fields versus other fields of fixed quadratic resolvent seen in the work of Bhargava–Shnidman and Cohen–Morra. I'll also discuss related results such as the fact that the shape is a complete invariant of complex cubic fields.

GUILLERMO MANTILLA-SOLER, Universidad de los Andes
A characterization of arithmetic equivalence via Galois representations

Let K be a number field and let $\zeta_K(s) = \displaystyle\sum_{n=0}^{\infty} \frac{a_n(K)}{n^s}$ be its Dedekind zeta function. Motivated by Tate's isogeny theorem we show that $\zeta_K(s)$ is completely determined by $a_\ell(K)$ for ℓ prime, and we show how these ideas could lead to new results on arithmetic equivalence.

AMALIA PIZARRO-MADARIAGA, Universidad de Valparaíso

Rational Products of Singular Moduli

The numbers of the form $j(\tau)$, where τ is an imaginary quadratic number with $Im(\tau) > 0$ and j is the j-invariant are called singular moduli. In this talk, we will show that with "obvious" exceptions the product of two singular moduli cannot be a non-zero rational number.

PEDRO LUIS DEL ÁNGEL RODRÍGUEZ, Centro de Investigación en Matemáticas A.C.

Eichler-Shimura and extensions of Hodge structures

Given a flat family of elliptic curves parametrized by a projective curve T, Peter Stiller associated to every element of the field K(T) a second order differential equation which is related to an extension of Hodge structures. Particularly interesting is the case when T is itself an elliptic curve.

We are interested in understanding the extensions of Hodge structures that arise in a similar fashion for flat families of Calabi-Yau varieties parametrized by Shimura varieties.

ADRIANA SALERNO, Bates College

Alternate Mirror Families and Hypergeometric Functions

Mirror symmetry predicts surprising geometric correspondences between distinct families of algebraic varieties. In some cases, these correspondences have arithmetic consequences. Among the arithmetic correspondences predicted by mirror symmetry are correspondences between point counts over finite fields, and more generally between factors of their Zeta functions. In this talk, we will present closed formulas for the point counts for our alternate mirror families of K3 surfaces and their relation to their Picard–Fuchs equations. Finally, we will discuss how all of this relates to hypergeometric functions and hypergeometric motives. This is joint work with: Charles Doran (University of Alberta, Canada), Tyler Kelly (University of Cambridge, UK), Steven Sperber (University of Minnesota, USA), John Voight (Dartmouth College, USA), and Ursula Whitcher (American Mathematical Society, USA).

CECILIA SALGADO, UFRJ

Rank bounds on fibrations of jacobians varieties

Let k be a number field, X a surface defined over k and $\pi : X \to B$ a fibration of genus g curves. Néron-Silverman Specialization Theorem states that the ranks of the Mordell-Weil groups of the generic and special fibers of the associated jacobian fibration satisfy $rk(J_t(k)) \geq rk(J_\eta(k(B)))$ for almost all t.

I will discuss work in progress towards showing that the above inequality is strict for infinitely many $t \in B(k)$, i.e., such that $rk(J_t(k)) \geq rk(J_\eta(k(B))) + 1$. This is motivated by preceding work on the case $g = 1$. This is work in progress with M. Hindry and A. Pacheco.

RICARDO TOLEDANO, Universidad Nacional del Litoral

S-minimal value set polynomials and towers of Garcia, Stichtenoth and Thomas type

An interesting family of tamely ramified recursive towers of function fields over finite fields was defined by Garcia, Stichtenoth and Thomas in 1997. They gave sufficient conditions to have asymptotically good towers in this family and all the examples were given over non prime fields. Later in 2001 H. Lenstra found a polynomial identity which explained why their conditions failed in the case of prime fields. In this talk we will show that a modification of Lentra's identity will allow us to relate the equations defining the towers in this family with the theory of minimal value set polynomials.

CHRISTELLE VINCENT, University of Vermont

Constructing hyperelliptic curves of genus 3 whose Jacobians have CM

For cryptographic applications, it is convenient to, given a CM field, be able to construct an abelian variety defined over the complex numbers with complex multiplication by an order in the ring of integers of that field.

It is currently well-understood how to do this in dimension 1, and a lot of progress has been done in dimension 2. We discuss here the challenges of constructing an abelian threefold with complex multiplication by the ring of integers of a sextic CM field and the work that has been done recently in this direction.

URSULA WHITCHER, Mathematical Reviews

Zeta functions of alternate mirror Calabi-Yau pencils

We prove that if two Calabi-Yau invertible pencils in projective space have the same dual weights, then they share a common polynomial factor in their zeta functions related to a hypergeometric Picard-Fuchs differential equation. The polynomial factor is defined over the rational numbers and has degree greater than or equal to the order of the Picard-Fuchs equation. This talk describes joint work with Charles Doran, Tyler Kelly, Adriana Salerno, Steven Sperber, and John Voight.