**Advances in Arrangement Theory**
**Progrès en théorie des arrangements**
(Org: **Jeremiah Bartz** (University of North Dakota), **Christin Bibby** (University of Western Ontario) and/et **Max Wakefield** (US Naval Academy))

**TAKURO ABE**, Kyushu University
*Hyperplane arrangements and Hessenberg varieties*

It is well-known by Borel that the coholomogy ring of the flag variety is isomorphic to the coinvariant algebra of the Weyl group. On the other hand, K. Saito's proof for the freeness of the Weyl arrangement enables us to re-construct the coinvariant algebra in terms of logarithmic vector fields, without Weyl groups. We generalize this result to a presentation of the coholomogy ring of Hessenberg varieties in terms of the logarithmic vector fields of ideal arrangements, which are known to be free. Moreover, we give several consequenses of this presentation.

**PAULINE BAILET**, Bremen University
*A vanishing result for the first twisted cohomology of affine varieties and applications to line arrangements*

(Joint work with A. Dimca and M. Yoshinaga)

Let $S$ be a smooth proper complex variety of dimension $\geq 2$ and $D = \sum_{i=1}^{n} D_i$ a divisor on $S$ ($D_i$ irreducible). Consider a rank one local system $\mathcal{L}$ on $U = S \setminus D$, with monodromy $t_i \in \mathbb{C}^{\times}$ around $D_i$. We give a general vanishing result for the first twisted cohomology group $H^1(U, \mathcal{L})$, generalizing a result due to Cohen-Dimca-Orlik. Then we give some applications in the context of hyperplane arrangement, namely local system cohomology of line arrangement complements. In particular, we will apply our result to determine the monodromy action on the Milnor fiber of two hyperplane arrangements: the Ceva arrangement and the exceptional reflection arrangement of type $G_{31}$.

**JEREMIAH BARTZ**, University of North Dakota
*Induced and Complete Multinets*

Multinets are certain configurations of lines and points with multiplicities in the complex projective plane. They appear in the study of resonance and characteristic varieties of complex hyperplane arrangement complements and cohomology of Milnor fibers. We investigate two properties of multinets, inducibility and completeness, and the relationship between with several examples presented. The main result is the classification of complete 3-nets.

**ANDREW BERGET**, Western Washington University
*Internal zonotopal algebras and monomial reflection groups*

The *internal zonotopal algebra* of an arrangement $\mathcal{A} \subset V$ is a zero (Krull) dimensional algebra that arises in the study of multivariate box splines. Its vector space dimension is the number of bases of the associated matroid with zero internal activity. When $\mathcal{A}$ is fixed by the action of a group $G \subset GL(V)$ the internal zonotopal algebra of the Gale dual $\mathcal{A}^{\perp}$ is a representation of $W$. In this talk I will present some results on this representation in the case when $W$ is one of the monomial reflection groups $G(m, 1, n)$.

**DANIEL C. COHEN**, Louisiana State University
*Residual freeness of arrangement groups*

A group $G$ is said to be residually free if every nontrivial element of $G$ is detected by a map to a free group. We investigate the residual freeness of arrangement groups, fundamental groups of complements of complex hyperplane arrangements.

This is work in progress with Mike Falk.

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**EMANUELE DELUCCHI**, Université de Fribourg (Switzerland)
*Fundamental polytopes of metric trees via hyperplane arrangements*

The problem of a combinatorial classification of finite metric spaces via their *fundamental polytopes* was suggested by Vershik in 2010, but to date the structure of these polytopes (even their face numbers) remains largely unknown. In this talk I will explain how to associate a hyperplane arrangement associated to every split pseudometric and how to use the combinatorics of the underlying matroid in order to compute the face numbers of fundamental polytopes and Lipschitz polytopes of tree-like metrics. I will discuss how to apply our results to specific examples and, time permitting, I will briefly comment on the potential of our model beyond tree-like metrics. This is joint work with Linard Hoessly.

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**GRAHAM DENHAM**, University of Western Ontario
*Local systems on complements of smooth hypersurface arrangements*

I will present joint work with Alex Suciu, where we give a simple sufficient condition for the complement of a smooth, complex arrangement of hypersurfaces to have certain cohomological vanishing properties. In particular, using recent work of De Concini and Gaiffi, we arrive at a uniform proof that the complements of linear, toric and elliptic hyperplane arrangements are both duality and abelian duality spaces. As a consequence, we see that the characteristic varieties and resonance varieties of such arrangements "propagate".

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**MICHAEL J. FALK**, Northern Arizona University
*Weak orders - left and right - and configuration spaces*

We describe a small acyclic category, generated by the left and right weak orders on a finite-type Coxeter group, whose nerve has the homotopy type of the associated unlabelled configuration space. The construction is based on a model of the circle action on complexified arrangement complements developed by the presenter with Emanuele Delucchi. This is joint work in progress with Dana Ernst and Sonja Riedel.

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**NIR GADISH**, The University of Chicago
*Finitely generated sequences of arrangements and representation stability*

Many linear subspace arrangements occur naturally in sequences that can be described succinctly, e.g. all braid arrangements are given by equations of a single form: "$z_i = z_j$". This situation can be formalized by the framework of a finitely generated $\mathfrak{C}$-diagram of arrangements, where $\mathfrak{C}$ is some indexing category. A fundamental result in this context is that, under certain structural assumptions on $\mathfrak{C}$, the intersection posets of a finitely generated sequence of arragnements exhibits a form of combinatorial stability, which in turn implies that the arrangements' complements exhibit cohomological representation stability. My talk will present the terminology mentioned above and applications of the observed representation stability.

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**BARBARA GUTIERREZ**, CINVESTAV
*"Secuencial topological complexity of the complement of complex hyperplane arrangements in general position".*

The secuencial topological complexity of a path connected space, is a concept introduced by Rudyak in 2010 as a generalization of the standard Farber's topological complexity. This concept is a homotopy invariant that measures the instabilities of the secuencial motion planning problem. In this talk, we will study the behavior of this invariant for subcomplexes of products of spheres, spaces closely related to the complement of complex hyperplane arrangements in general position.

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**DANIEL MOSELEY**, Jacksonville University
*The Orlik-Terao algebra and the cohomology of configuration space*

The cohomology of the configuration space of $n$ points in $\mathbb{R}^3$ is isomorphic to the regular representation of the symmetric group, which acts by permuting the points. We give a proof of this fact by showing that the cohomology ring is canonically isomorphic to the associated graded of the cohomology of the configuration space of $n$ points in $\mathbb{R}^1$ with the Varchenko-Gelfand filtration. Also, we give a recursive algorithm for computing the Orlik-Terao algebra of the Coxeter arrangement of type $A_{n-1}$ as a graded representation of $S_n$, and we give a conjectural description of this representation in terms of the cohomology of the configuration space of $n$ points in $\mathrm{SU}(2)$ modulo translation.

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**JORGE PEREIRA**, IMPA
*Representations of quasi-projective groups*

The talk will review results describing representations of fundamental groups of quasi-projective manifolds on $\mathrm{Aff}(\mathbb{C})$, $\mathrm{SL}(2,\mathbb{C})$, and $\mathrm{Diff}(\mathbb{C},0)$.

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**RITA JIMENEZ ROLLAND**, Universidad Nacional Autónoma de México
*Stability for hyperplane complements and statistics over finite fields*

In this talk I will discuss a remarkable relationship between the cohomology of hyperplane complements of type A and B/C and the combinatorics of spaces of square free polynomials over a finite field. I will describe the underlying algebraic structure of these cohomology algebras that implies asymptotic stability for statistics over finite fields. This is joint work with Jennifer Wilson.

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**ALEXANDER I. SUCIU**, Northeastern University
*Arrangement complements and Milnor fibrations*

I will discuss some recent advances in our understanding of the multiple connections between the combinatorics of an arrangement of hyperplanes, the topology of its complement and boundary manifold, and the monodromy of its Milnor fibration.

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**HIROAKI TERAO**, Hokkaido University
*On the Exponents of Restrictions of Weyl Arrangements*

It has been known that every restriction of an arbitrary Weyl arrangement is a free arrangement due to P. Orlik - H. Terao (1993) and T. Hoge - G. Roehrle (2013). In this talk we will discuss how their exponents are determined. The discussion is closely related to the divisionally freeness theorem proved by T. Abe (2016).

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**MASAHIKO YOSHINAGA**, Hokkaido University
*Remarks on characteristic quasi-polynomials of deformed Weyl arrangements*

Recently it is found that the characteristic quasi-polynomial of certain deformations of the Weyl arrangement can be expressed in terms of Eulerian polynomials and Ehrhart quasi-polynomial of the fundamental alcove. This has been applied to prove the so-called "functional equation" of the characteristic polynomial of Linial arrangements. We will discuss further applications of the formula.