Advances in Analysis, PDE's and Related Applications
Progrès en analyse, ÉDP et applications connexes
(Org: **Marcia Federson** (University of San Carlos), **Tepper L Gill** (Howard University) and/et **Erik Talvila** (University of Fraser Valley))

GEORGE CHEN, Cape Breton University
Global Existence for a Singular Gierer-Meinhardt and Enzyme Kinetics System

In this talk we discuss the existence results for a singular system subject to zero Dirichlet boundary conditions, which originally arose in studies of pattern-formation in biology and chemical reactions in Chemistry. The mathematical difficulties are that the system becomes singular near the boundary and it lacks a variational structure. We use a functional method to obtain both upper and lower bounds for the perturbed system and then use Sobolev embedding theorem to prove the existence of a pair of positive solutions under suitable conditions. This method is first used in a singular parabolic system and is completely different than the traditional methods of sub and super solutions.

MARCIA CRISTINA A. B. FEDERSON, Universidade de Sao Paulo
The Generalized Feynman Integral and Applications

The aim of this talk is to present the generalized Feynman integral, also known as the Henstock integral, and some of its features and applications to quantum mechanics and finance.

IRENE FONSECA, Carnegie Mellon University
Variational Models for Image Processing

The mathematical treatment of image processing is strongly hinged on variational methods, partial differential equations, and machine learning. The bilevel scheme combines the principles of machine learning to adapt the model to a given data, while variational methods provide model-based approaches which are mathematically rigorous, yield stable solutions and error estimates. The combination of both leads to the study of weighted Ambrosio-Tortorelli and Mumford-Shah variational models for image processing.

MARIANA SMIT VEGA GARCIA, University of Washington
The singular free boundary in the Signorini problem

In this talk I will overview the Signorini problem for a divergence form elliptic operator with Lipschitz coefficients, and I will describe a few methods used to tackle two fundamental questions: what is the optimal regularity of the solution, and what can be said about the singular free boundary in the case of zero thin obstacle. The proofs are based on Weiss and Monneau type monotonicity formulas. This is joint work with Nicola Garofalo and Arshak Petrosyan.

KATHRYN HARE, University of Waterloo
Local dimensions of self-similar measures with overlap

The local dimension of a measure is a way to quantify its local behaviour. For example, the local dimension of Lebesgue measure is one everywhere, reflecting the fact that it is uniform in its concentration. For self-similar measures that satisfy a suitable separation condition, it is well known that the set of attainable local dimensions is a closed interval, but for measures which fail to satisfy this condition the situation is more complicated and unclear. In this talk we will discuss a general theory for a class of measures with overlap, which includes interesting examples such as Bernoulli convolutions and convolutions of Cantor measures, and we will see that different phenomena can arise.

SONIA MAZZUCCHI, Math Department, University of Trento (Italy)
Projective systems of functionals and generalized Feynman-Kac formulae

The construction of a functional integral representation for the solution of the Schroedinger equation, in other words the mathematical definition of "Feynman path integrals", requires an integration theory on infinite dimensional spaces which extends the Lebesgue one. In this talk I shall introduce a generalized approach to infinite dimensional integration which includes, on one hand, both probabilistic and oscillatory integrals and provides, on the other hand, the mathematical basis for the construction of generalized Feynman-Kac formulae. In particular it can be applied to the representation of the solution to partial differential equations which do not satisfy a maximum principle, such as, for instance, the Schödinger equation or N-order heat-type equations of the form

$$\begin{aligned} \frac{\partial}{\partial t}u(t,x) &= a\frac{\partial^N}{\partial x^N}u(t,x), \\ u(0,x) &= f(x) \end{aligned}$$

where $t \in \mathbb{R}^+, x \in \mathbb{R}, a \in \mathbb{C}$ and $N \in \mathbb{N}$.

LUIS ANGEL GUTIERREZ MENDEZ, Emeritus Autonomous University of Puebla
Space of successions classics contained in the space of Henstock-Kurzweil integrable functions

In this talk, we will exhibit a technique that shows that certain spaces of successions classic and not classic are contained, as copies, in the space of the Henstock-Kurzweil integrable functions. In particular, we will prove that the spaces c_0 and l_p are contained in the space of the Henstock-Kurzweil integrable functions.

JAQUELINE GODOY MESQUITA, University of Brasilia
Generalized ODEs and measure differential equations: results and applications

In this talk, we present results concerning prolongation of solutions, boundedness of solutions as well as stability results for generalized ODEs. Also, using the correspondence between the solutions of generalized ODEs and the solutions of measure differential equations, we extend our results for these last equations, obtaining more general results than the ones found in the literature. Finally, by the correspondence between measure differential equations and dynamic equations on time scales, we also prove our the results for these last ones.

TIMOTHY MYERS, Howard University
Lebesgue Integration on a Banach Space with a Schauder Basis

This talk will feature the construction of a Lebesgue measure and integral on any Banach space $\mathcal{B}$ with a Schauder basis. This theory has the advantage that the integral is computable from below as a limit of Lebesgue integrals on Euclidean space as the dimension $n \to \infty$, so that we may evaluate infinite dimensional quantities by means of finite dimensional approximation. Applications will be discussed.

JUAN HECTOR ARREDONDO RUIZ, Universidad Autónoma Metropolitana
On the Factorization theorem in the space of Henstock-Kurzweil integrable functions

We apply the factorization theorem of Rudin and Cohen to the space of Henstock-Kurzweil integrable functions $HK(R)$. This implies a factorization for the isometric spaces A_C and B_C . We also study in this context the Banach algebra $HK(R) \cap BV(R)$, which is also a dense subspace of $L^2(R)$. This space is in some sene analogous to $L^1(R) \cap L^2(R)$. However, while $L^1(R) \cap L^2(R)$ factorizes as $L^1(R) \cap L^2(R) * L^1(R)$, via the convolution operation $*$, it will be shown that $HK(R) \cap BV(R) * L^1(R)$ is a Banach subalgebra of $HK(R) \cap BV(R)$. Joint work with Maria G. Morales

ERIK TALVILA, University of the Fraser Valley
The heat equation with the continuous primitive integral

A Schwartz distribution has a continuous primitive integral if it is the distributional derivative of a function that is continuous on the extended real line. This generalises the Lebesgue and Henstock–Kurzweil integrals. The Alexiewicz norm of f is $\|f\| = \sup |\int_I f|$ where the supremum is over all intervals $I \subset \mathbb{R}$. The space of distributions integrable in this sense is then a Banach space isometrically isomorphic to the continuous functions on the extended real line with the uniform norm. Many properties familiar from Lebesgue integration continue to hold for these distributions.

The one-dimensional heat equation is considered with initial data that is integrable in the sense of the continuous primitive integral. Let $\Theta_t(x) = \exp(-x^2/(4t))/\sqrt{4\pi t}$ be the heat kernel. With initial data f that is the distributional derivative of a continuous function, it is shown that $u_t(x) := u(x,t) := f * \Theta_t(x)$ is a classical solution of the heat equation $u_{11} = u_2$. The estimate $\|f * \Theta_t\|_\infty \leq \|f\|/\sqrt{\pi t}$ holds. The initial data is taken on in the Alexiewicz norm, $\|u_t - f\| \to 0$ as $t \to 0^+$. The solution of the heat equation is unique under the assumptions that $\|u_t\|$ is bounded and $u_t \to f$ in the Alexiewicz norm for some integrable f.

FRANCISCO JAVIER MENDOZA TORRES, Facultad de Cs. Físico Matemáticas, Benemérita Universidad Autónoma de Puebla
The Convolution Theorem over a subset of bounded variation functions

In this talk we prove the Convolution Theorem for the Fourier integral transform over a subset of bounded variation functions which is dense in $L^2(\mathbb{R})$. Moreover, we study some features of those bounded linear transformations T defined on that intersection with values in the space of bounded continuous functions on $\mathbb{R}$, for which the convolution identity $T(f*g) = Tf \cdot Tg$ holds.

ALEJANDRO VÉLEZ-SANTIAGO, University of Puerto Rico - Mayagüez
A quasi-linear Neumann problem of Ambrosetti-Prodi type in non-smooth domains

We investigate the solvability of the Ambrosetti-Prodi problem for the p-Laplace operator with Neumann boundary conditions. Using a priori estimates, regularity theory, a sub-supersolution method, and the Leray-Shauder degree theory, we obtain a necessary condition for the non-existence of solutions (in the weak sense), the existence of at least one solution, and the existence of at least two distinct solutions. Moreover,we establish global Hüeolder continuity for weak solutions of the Neumann problem of AmbrosettiProdi type on a large class of non-smooth domains.