Advances in Algebraic and Analytic Number Theory
Progrès en théorie des nombres algébrique et analytique
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OLGA BALKANOVA, University of Turku
Non-vanishing of automorphic L-functions in the weight aspect

In this talk, we show that the percentage of primitive cusp forms of level one and weight $4k \to \infty$, $k \in \mathbf{N}$ for which the associated L-function at the central point is no less than $(\log k)^{-2}$ is at least 20% for an individual weight and at least 50% on average. The key ingredients of our proof are the Kuznetsov convolution formula and the Liouville-Green method. This is a joint work with Dmitry Frolenkov.

EMANUEL CARNEIRO, IMPA - Rio de Janeiro
Fourier optimization problems in number theory

Fourier optimization problems appear naturally within several different fields of mathematics, particularly in analysis and number theory. These are problems in which one imposes certain conditions on a function and its Fourier transform, and then wants to optimize a certain quantity. A recent example is given in the proof of the optimal sphere packing in dimensions 8 and 24. In this talk I want to show how certain optimization problems of this sort appear in the theory of the Riemann zeta-function, prime gaps and weighted inequalities.

AYLA GAFNI, University of Rochester
Pair correlation statistics in subsets of the integers

Given $\mathcal{A} \subset \mathbb{N}$ and $\alpha \in \mathbb{R}$, it is often of interest to consider pair correlations of the set $\alpha\mathcal{A}$ and their distribution modulo 1. Denote by A_N the first N elements of $\mathcal{A}$. We say that $\mathcal{A}$ is "metric Poissonian" if

$$\frac{1}{N}\sum_{\substack{a,b\in A_N\\ a\neq b}} 1_{[-s/N,s/N]}(\{\alpha(a-b)\}) \to 2s \quad \text{as} \quad N\to\infty,$$

for almost all α and for all fixed s, where $\{x\} = x - \lfloor x \rfloor$ denotes the fractional part of x. The metric Poissonian property is a stronger notion of equidistibution modulo 1, and is closely related to the additive energy of the set. Indeed, Aistleitner, Larcher, and Lewko have shown that if the additive energy satisfies $E(A_N) = O(N^{3-\varepsilon})$ then $\mathcal{A}$ is metric Poissonian. In an appendix to the same paper, Bourgain gives that $\mathcal{A}$ cannot be metric Poissonian if $\limsup_{n\to\infty} E(A_N)N^{-3} > 0$. In this talk, we will discuss ways in which density and additive energy can be used to determine whether a set $\mathcal{A} \subset \mathbb{N}$ has the metric Poissonian property. This is joint work with Thomas Bloom, Sam Chow, and Aled Walker.

VICTOR CUAUHTÉMOC GARCIA, Universidad Autonoma Metropolitana - Azcapotzalco
Additive basis with coefficients of newforms

Let $f(z) = \sum_{n=1}^{\infty} a(n)e^{2\pi i n z}$ be a normalized Hecke eigenform in $S_{2k}^{\mathrm{new}}(\Gamma_0(N))$ with integer Fourier coefficients.
In this talk, we prove that there exists a constant $C(f) > 0$ such that any integer is a sum of at most $C(f)$ coefficients $a(n)$.

LEO GOLDMAKHER, Williams College
The Pólya-Vinogradov Inequality

The famous Pólya-Vinogradov inequality for character sums was proved a century ago, but for the past sixty years it's been overshadowed by a different character sum bound due to Burgess. In this talk I will try to explain why you should still care about Pólya-Vinogradov.

JHON JAIRO BRAVO GRIJALBA, Universidad del Cauca
Linear forms in k-Fibonacci sequences

For an integer $k \geq 2$, we consider the $k-$Fibonacci sequence $(F_n^{(k)})_n$ which starts with $0, \ldots, 0, 1$ (k terms) and each term afterwards is the sum of the k preceding terms. In this talk, we report about some arithmetic properties of $(F_n^{(k)})_n$ and study some Diophantine equations involving $k-$Fibonacci numbers. This is a joint work with Carlos Gómez and Florian Luca.

PIPER HARRON, University of Hawaii at Manoa
Shapes of Galois Quartic Number Fields

It is known that the shapes of S_4-quartic number fields are equidistributed in the space of shapes of rank 3 lattices. What happens if we restrict ourselves to Galois quartics? The Galois automorphisms force the shapes to live in lower-dimensional subspaces. We determine the shapes of Galois quartic fields, finding that where they lie depends on the Galois group and the ramification of 2. We also study the distribution of these shapes in these subspaces. This work is joint with Robert Harron.

SUN KIM, University of Illinois at Urbana-Champaign
Sums of squares and Bessel functions

In 1934, the Russian mathematician A. I. Popov stated, but did not rigorously prove, a beautiful series transformation involving $r_k(n)$ and certain Bessel functions. We provide a proof of this identity for the first time, as well as for another identity, which can be regarded as both an analogue of Popov's identity and an identity involving $r_2(n)$ from Ramanujan's lost notebook. Furthermore, we establish a new transformation between a series consisting of $r_k(n)$ and a product of two Bessel functions, and a series involving $r_k(n)$ and the Gaussian hypergeometric function. This transformation can be considered as a massive generalization of well-known results of G. H. Hardy, and of A. L. Dixon and W. L. Ferrar. This is joint work with B. C. Berndt, A. Dixit and A. Zaharescu.

LUIS LOMELI, Instituto de Matemáticas PUCV
Asai cube L-functions and the local Langlands correspondence

(Joint with Guy Henniart). Let F be a non-Archimedean locally compact field, and let E be a cubic separable extension of F. Let $\mathbf{H}$ be a simply connected quasi-split semisimple group over F of type D_4, with triality corresponding to E, and let $\mathbf{L}$ be its Levi subgroup with derived group $\mathrm{Res}_{E/F}\mathrm{SL}_2$. To any irreducible smooth generic representation π of $\mathrm{GL}_2(E)$, the Langlands-Shahidi method applied to $(\mathbf{H}, \mathbf{L})$ attaches an Asai cube L-function and related local factors. If σ is the Weil-Deligne representation corresponding to π via the Langlands correspondence, we prove that Asai cube local factors for π are the local factors for the Weil-Deligne representation obtained from σ via tensor induction from E to F. A consequence is that Asai cube γ- and ε-factors become stable under twists by highly ramified characters.

AMALIA PIZARRO MADARIAGA, Universidad de Valparaíso
Irreducible characters with bounded root Artin conductor

Let K be an algebraic number field such that $K/\mathbb{Q}$ is Galois and let χ be the character of a linear representation of $Gal(K/\mathbb{Q})$. The Artin conductor f_χ of χ is given by

$$f_\chi = \prod_{p \nmid \infty} p^{f_p(\chi)}$$

with

$$f_p(\chi) = \frac{1}{|G_0|} \sum_{j \geq 0} (|G_j|\chi(1) - \chi(G_j)),$$

where G_i is the i-th ramification group of $K_{\mathfrak{b}}/\mathbb{Q}_p$ with $\mathfrak{b}$ a prime over p and $\chi(G_j) = \sum_{g \in G_j} \chi(g)$.
In this talk, we will prove that the growth of the Artin conductor is at most, exponential in the degree of the character.

NATHAN NG, University of Lethbridge
The sixth moment of the Riemann zeta function and ternary additive divisor sums

Hardy and Littlewood initiated the study of the 2k-th moments of the Riemann zeta function on the critical line. In 1918 Hardy and Littlewood established an asymptotic formula for the second moment and in 1926 Ingham established an asymptotic formula for the fourth moment. In this talk we consider the sixth moment of the zeta function on the critical line. We show that a conjectural formula for a certain family of ternary additive divisor sums implies an asymptotic formula for the sixth moment. This builds on earlier work of Ivic and of Conrey-Gonek.

ARINDAM ROY, Rice University
On the distribution of imaginary parts of zeros of derivatives of the Riemann ξ-function

Let $\xi(s)$ be the completed Riemann zeta-function. It is known that the Riemann hypothesis of $\xi(s)$ implies the Riemann hypothesis of $\xi^{(m)}(s)$, where m is a positive integer. We investigate some results on the distribution of imaginary parts of zeros of $\xi^{(m)}(s)$. We also obtain a zero density result of $\xi^{(m)}(s)$.

DAMARIS SCHINDLER, Utrecht University
On integral points on degree four del Pezzo surfaces

We report on our investigations concerning algebraic and transcendental Brauer-Manin obstructions to integral points on complements of a hyperplane section in degree four del Pezzo surfaces. This is joint work with Joerg Jahnel.

ARI SHNIDMAN, Boston College
Quadratic twists of an elliptic curve admitting a 3-isogeny

I'll present joint work with Manjul Bhargava, Zev Klagsbrun, and Robert Lemke Oliver. Let E be an elliptic curve over a number field, and assume E has a rational subgroup of order 3. We prove that as you vary over all quadratic twists of E, the average rank of these twists is bounded. Over $\mathbb{Q}$, we further show that a positive proportion of twists have rank 0 and, assuming finiteness of Sha, that a positive proportion have rank 1. We also construct many twist families with a large proportion of twists having large Sha. The bounds in these results depend on the reduction types of the curves and are completely explicit.

AMANDA TUCKER, University of Rochester
Statistics of genus numbers of cubic fields

The genus number of a number field is the degree of the maximal unramified extension of the number field that is obtained as a compositum of the field with an abelian extension of Q. We will explain our proof that 96.2% of cubic fields have genus number one and, if time permits, talk about some applications. This represents joint work with Kevin McGown.

AKSHAA VATWANI, University of Waterloo
Twin primes and the parity problem

We relate the twin prime problem to a conjecture of Chowla type regarding the Möbius function. This is joint work with Professor Ram Murty.