Majorization in Von Neumann Algebras

A classical result in matrix theory characterizes the convex hull of the unitary orbit of a self-adjoint matrix using spectral data. The description of these convex hulls has many applications such as characterizing the possible diagonal $n$-tuples of a self-adjoint matrix based on its eigenvalues. As all of these problems have natural analogues in an arbitrary unital $C^*$-algebra, it is natural to ask whether we can generalize these results.

In this talk, we discuss extensions of these results to von Neumann algebras. This includes characterizations of diagonal of operators onto maximal abelian self-adjoint algebras (joint work with M. Kennedy). In addition, using von Neumann algebras, we characterize the norm-closed convex hulls of the unitary orbits of self-adjoint operators in any unital $C^*$-algebra (joint work with P. Ng and L. Robert).