Explicit sentences distinguishing McDuff’s II$_1$ factors

McDuff was the first to give a family of continuum many pairwise nonisomorphic separable II$_1$ factors. In a recent paper by Boutonnet, Chifan, and Ioana, it was shown that the elements of this family also have pairwise nonisomorphic ultrapowers. As a result, none of these factors are elementarily equivalent, meaning that for every pair of distinct elements $M_\alpha$ and $M_\beta$ of this family, there is some sentence $\sigma$ such that the value of $\sigma$ in $M_\alpha$ differs from the value of $\sigma$ in $M_\beta$. However, at first glance, it was not clear how to extract such sentences from their proof. In joint work with Bradd Hart, we used Ehrenfeucht-Fraisse games to give an upper bound on the complexity of sentences distinguishing the McDuff factors. In this talk, I will discuss recent joint work with Hart and Henry Towsner, where we show how a finer analysis of the Boutonnet-Chifan-Ioana result can be used to write down explicit sentences distinguishing the McDuff factors.