We will start the talk with an overview of (one-sided) shift spaces over infinite alphabets. Then we will explain the notion of ultragraphs, with generalize directed graphs, and use these combinatorial objects to define a notion of (one-sided) edge shift spaces (which, coincides with the edge shift space of a graph). We then go on to show that these shift spaces have some nice properties, as metrizability and basis of compact open sets. To finalize we examine shift morphisms between these shift spaces: we give an idea how to show that if two (possibly infinite) ultragraphs have edge shifts that are conjugate, via a conjugacy that preserves length, then the associated ultragraph $C^*$-algebras are isomorphic.