An affine quantum cohomology ring

A theorem of B. Kim identified the relations of the quantum cohomology ring of the (generalized) flag manifolds with the conserved quantities for the Toda lattice. It is expected that a similar statement exists, relating a quantum cohomology ring for the affine flag manifolds to the periodic Toda lattice. I will show how to construct a deformation of the usual quantum cohomology ring, depending on an additional affine quantum parameter. It turns out that the conserved quantities of the (dual) periodic Toda lattice give the ideal of relations in the new ring. The construction of the ring multiplication involves the "curve neighborhoods" of Schubert varieties in the affine flag manifold. For ordinary flag manifolds, these were defined and studied earlier by the speaker in several joint works with A. Buch, P.E. Chaput, and N. Perrin. This is joint with Liviu Mare.