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Operations sufficient to obtain any Pogorelov polytope from barrels. Improvements for fullerenes.

A fullerene is a simple 3-polytope with all facets pentagons and hexagons. Any fullerene is a Pogorelov polytope, i.e. it can be realized in Lobachevski (hyperbolic) 3-space as a bounded right-angled polytope. A $k$-barrel is a simple 3-polytope with boundary glued from two patches consisting of a $k$-gon surrounded by pentagons. The 5-barrel is the dodecahedron. Results by T. Inoue (2008) imply that any Pogorelov polytope can be combinatorially obtained from $k$-barrels by a sequence of $(s, k)$-truncations (cutting off $s$ consequent edges of a $k$-gon by a single plane), $2 \leq s \leq k - 4$, and connected sums along $k$-gonal faces (combinatorial analog of glueing two polytopes along $k$-gons perpendicular to adjacent facets). We prove that any Pogorelov polytope except for $k$-barrels can be obtained from the 5- or 6-barrel by $(2, k)$-truncations, $k \geq 6$, and connected sums with 5-barrels along pentagons. In the case of fullerenes we prove a stronger result. Let $(2, k; m_1, m_2)$-truncation be a $(2, k)$-truncation that cuts off two edges intersecting an $m_1$-gon and an $m_2$-gon by vertices different from the common vertex. There is an infinite family of connected sums of 5-barrels along pentagons surrounded by pentagons called $(5, 0)$-nanotubes. We prove that any fullerene except for the 5-barrel and the $(5, 0)$-nanotubes can be obtained from the 6-barrel by a sequence of $(2, 6; 5, 5)$-, $(2, 6; 5, 6)$-, $(2, 7; 5, 5)$-, $(2, 7; 5, 6)$-truncations such that all intermediate polytopes are either fullerenes or Pogorelov polytopes with facets 5-, 6- and one 7-gon with the heptagon adjacent to some pentagon. This work is supported by the Russian Science Foundation under grant 14-11-00414.