NIKOLAI EROKHOVETS, Steklov Mathematical Institute of Russian Academy of Sciences
Operations sufficient to obtain any Pogorelov polytope from barrels. Improvements for fullerenes.

A fullerene is a simple 3-polytope with all facets pentagons and hexagons. Any fullerene is a Pogorelov polytope, i.e. it can be realized in Lobachevski (hyperbolic) 3-space as a bounded right-angled polytope. A k-barrel is a simple 3-polytope with boundary glued from two patches consisting of a k-gon surrounded by pentagons. The 5-barrel is the dodecahedron. Results by T. Inoue (2008) imply that any Pogorelov polytope can be combinatorially obtained from k-barrels by a sequence of (s,k)-truncations (cutting off s consequent edges of a k-gon by a single plane), $2 \leq s \leq k-4$, and connected sums along k-gonal faces (combinatorial analog of glueing two polytopes along k-gons perpendicular to adjacent facets). We prove that any Pogorelov polytope except for k-barrels can be obtained from the 5- or 6-barrel by $(2,k)$-truncations, $k \geqslant 6$, and connected sums with 5-barrels along pentagons. In the case of fullerenes we prove a stronger result. Let $(2,k;m_1,m_2)$-truncation be a $(2,k)$-truncation that cuts off two edges intersecting an m_1-gon and an m_2-gon by vertices different from the common vertex. There is an infinite family of connected sums of 5-barrels along pentagons surrounded by pentagons called $(5,0)$-nanotubes. We prove that any fullerene except for the 5-barrel and the $(5,0)$-nanotubes can be obtained from the 6-barrel by a sequence of $(2,6;5,5)$-, $(2,6;5,6)$-, $(2,7;5,5)$-, $(2,7;5,6)$-truncations such that all intermediate polytopes are either fullerenes or Pogorelov polytopes with facets 5-, 6- and one 7-gon with the heptagon adjacent to some pentagon. This work is supported by the Russian Science Foundation under grant 14-11-00414.