Spaces all of whose loops are small

There is a long history of trying to generalize covering spaces. A recent flurry of work on the subject has revived interest. There are spaces which are an obstacle in this study, namely, those spaces having the property that all loops are small. These spaces have no covering spaces. They deserve special consideration since any theory of generalized covering spaces must take them into account.

In this talk we show that if $G$ is any group, then there is a pointed space $(X, x_0)$ having the property that all loops are small and such that $\pi_1(X, x_0) \cong G$. The space $X$ that we construct is not metrizable. In fact, it cannot be metrizable in general since there is no space $(X, x_0)$ with all loops small such that $X$ is first-countable at $x_0$ and with $\pi_1(X, x_0) \cong \mathbb{Z}$. On the other hand, we have other constructions of such spaces for certain groups $G$ for which $X$ is metrizable.

We will elaborate on these results and relate them to other work being done on generalized covering spaces.