
DANUTA KOŁODZIEJCZYK, Warsaw University of Technology
Cartesian Powers of Shapes of FANR's and Polyhedra

The Cartesian product $\text{Sh}(X) \times \text{Sh}(Y)$ of the shapes of compacta X and Y is defined as $\text{Sh}(X \times Y)$. Then $\text{Sh}(X)$ and $\text{Sh}(Y)$ are called factors of $\text{Sh}(X \times Y)$. Similarly one defines product and factors of pointed shapes of compacta, and factors in the homotopy category.

We prove that, if $(X, x) \in \text{FANR}$ and $\text{Sh}^n(X, x) = \text{Sh}(X, x)$, for some $2 \leq n \in \mathbb{N}$, then $\text{Sh}(X, x) = 1$. This resolves positively a problem of J. Dydak, A. Kadlof, S. Nowak [3]. Furthermore, if $(X, x) \in \text{FANR}$, then (X, x) cannot be a proper factor of itself.

The same results we get for polyhedra in the homotopy category of CW -complexes. (In particular, on ANR 's shape and homotopy theory coincide.)

Thus, the answer to the following question of K. Borsuk [2] is positive: Is it true that if $X \in \text{ANR}$ and $\text{Sh}^n(X, x) = \text{Sh}(X, x)$, for some $2 \leq n \in \mathbb{N}$, then $X \in \text{AR}$? An equivalent problem was also published in [1, Problem (7.13), p. 142].

Some related results and open problems in the homotopy category of CW -complexes, in the shape category of compacta, and on finitely presented groups, are also discussed.

REFERENCES

- [1] K. Borsuk, *Theory of Shape*, Polish Scientific Publishers 59, Warsaw, 1975.
- [2] K. Borsuk, *Theory of Shape*, Lecture Notes Series, 28. Matematisk Institut, Aarhus Universitet, Aarhus, 1971.
- [3] J. Dydak, A. Kadlof, S. Nowak, *Open Problems in Shape Theory*, University of Warsaw, 1981.