Hausdorffization of the space of equivalence classes

Let $X$ and $M$ are connected and locally path connected Hausdorff topological spaces (so they are path connected) and let $F : X \to M$ be a continuous mapping such that for every $x \in M$ the set $F^{-1}(x)$ is locally path connected. We introduce on $X$ an equivalence relation: $x \sim y$ if $x$ and $y$ belong to the same connected component of $F^{-1}(x)$ and denote the quotient $X/\sim$ by $X_F$ endowed with the quotient topology. In general, the space $X_F$ need not to be Hausdorff even in simple situations. For example, let $X$ be the strip $\{(x, y) : -2 \leq y \leq 2\}$ in the plane with the ray $\{x \geq 0, y = 0\}$ cut out, $M$ is the $x$-axis and $F(x, y) = x$.

There are several examples, pertinent to complex analysis, where $X_F$ is seemingly non-Hausdorff. In our talk we will describe an algorithm that reduces $X_F$ to a Hausdorff space without the loss of information and demonstrate how it can be applied to known examples.