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Motivic Virtual Fundamental Classes

Let B be a reasonable base-scheme and Z a quasi-projective B-scheme. Relying on the Grothendieck 6-functor formalism for the motivic stable homotopy category, we define an object $C^{st}_{Z/B}$ in the motivic stable homotopy category $\mathsf{SH}(B)$, which we call the *intrinsic stable normal cone* of Z over B. For a motivic ring spectrum $\mathcal{E}$, we construct a fundamental class $[C^{st}_{Z/B}]_{\mathcal{E}}$ in $\mathcal{E}^{0,0}(C^{st}_{Z/B})$ and use this to construct for each perfect obstruction theory $\phi : E \to L_{Z/B}$ a virtual fundamental class $[Z,\phi]^{vir}_{\mathcal{E}} \in \mathcal{E}^{0,0}(\pi_{Z!}\Sigma^{E^\vee}1_Z)$. Here $\pi_Z : Z \to B$ is the structure morphism and we assume that B is affine. There are also G-equivariant versions of these constructions for G a "tame" algebraic group over B.

Taking $B = \mathrm{Spec}k$ and $\mathcal{E} = H\mathbb{Z}$, the spectrum representing motivic cohomology, we recover the definition of the fundamental class $[C_{Z/B}] \in \mathsf{CH}_0(C_{Z/B})$ of the intrinsic normal cone $C_{Z/B}$ of Z and the virtual fundamental class $[Z,\phi]^{vir} \in \mathsf{CH}_r(Z)$, $r = \mathrm{rank}E$, as defined by Behrend-Fantechi. Taking $\mathcal{E} = EM(K^{MW}_*)$, we get a virtual fundamental class $[Z,\phi]^{vir}_{K^{MW}_*} \in \tilde{\mathsf{CH}}_r(Z, \det^{-1}E)$, with $\tilde{\mathsf{CH}}$ the Chow-Witt theory of Barge-Morel and Fasel. In case $r = 0$, $\det E = \mathcal{O}_Z$, and Z projective over k, we can push this class forward to get a Grothendieck-Witt degree $\tilde{deg}[Z,\phi]^{vir}_{K^{MW}_*} \in \mathsf{GW}(k)$.